EC 610 EXAM 1 - Fall 2014 SOLUTIONS

1. a) Maximizing joint profit can lead to higher profits (FALSE)
   b) Homogeneous in (y, p_1, p_2). No need for homothetic in (p_1, p_2) (FALSE)
   c) From Roy's identity $dp_2/dp_1 = -\dfrac{-\partial V/\partial p_1 / \partial V/\partial y}{-\partial V/\partial p_2 / \partial V/\partial y} = -\dfrac{x_1^M}{x_2^m}$ (TRUE)
   d) If MCs are constant,
   with $MC_1 < MC_2 \Rightarrow$ Firm 1 sets $p = MC_2$, $\hat{\pi}_2 = 0$ (FALSE)

2. (a) $q_i = p/2 \Rightarrow q^s = \frac{3p}{2}$ (b) $\frac{3p}{2} = 10 - p \Rightarrow p^* = 4$

   (c) $\hat{\pi}_3 = 4 \times \left(\frac{4}{2}\right) - \left(\frac{4}{2}\right)^2 - 4 = 0$ $\hat{\pi}_5 = \hat{\pi}_6 = -1$

   Thus, 3 firms is the long-run equilibrium

3. (a) Need $P = 15$ $q_2 = 15$, then need $q_1 = 70$ (tied for best)

   (b) $\pi = (100 - q_1 - q_2)(q_1 + q_2) - 15q_1 - \frac{1}{2}q_2^2$

$$\Rightarrow \begin{cases} 100 - 2q_1 - 2q_2 - 15 = 0 \\ 100 - 2q_1 - 2q_2 - q_2 = 0 \end{cases} \qquad \begin{aligned} 85 - 2q_1 - 2q_2 &= 0 \\ 100 - 2q_1 - 3q_2 &= 0 \end{aligned}$$

$$15 - q_2 = 0 \qquad q_2^* = 15 \quad q_1^* = 27.5$$

$$p^* = 100 - 42.5 = 57.5$$

   (c) $\pi_1 = (100 - q_1 - q_2)q_1 - 15q_1$
   $\pi_2 = (100 - q_1 - q_2)q_2 - \frac{1}{2}q_2^2$

$$\Rightarrow \begin{cases} 85 - 2q_1^* - q_2 = 0 \\ 100 - q_1 - 3q_2^* = 0 \end{cases} \qquad q_1^* = \frac{85 - q_2}{2} \qquad q_2^* = \frac{100 - q_1}{3}$$

$$q_1 = 31 \quad q_2 = 23 \quad P = 46$$

4.

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4. Budget Constraint for exporting good 1

$$(p_1+s)(\bar{x}_1 - x_1) + p_2(\bar{x}_2 - x_2) = 0 \quad ; \text{where } \bar{x}_1 - x_1 > 0$$

$$\frac{\partial x_1}{\partial p_1} = \frac{\begin{vmatrix} 0 & -(\bar{x}_1 - x_1) & -p_2 \\ -(p_1+s) & \lambda & u_{12} \\ -p_2 & 0 & u_{22} \end{vmatrix}}{|H|} = \underbrace{-(\bar{x}_1 - x_1)\frac{\partial x_1}{\partial y}}_{\text{Pos inc effect}} \underbrace{- \frac{\lambda}{|H|} p_2^2}_{\text{Neg Sub Effect}} \gtrless 0$$

5. (a) $x_1 = x_2 = \frac{y}{p_1+p_2}$ $\quad V = \frac{y}{p_1+p_2} \quad \Rightarrow \quad e = u_0(p_1+p_2)$

(b) $x_1 = x_2 = \frac{y}{p_1+p_2}$ $\quad V = \frac{\sqrt{y}}{\sqrt{p_1+p_2}} \quad \Rightarrow \quad e = u_0^2(p_1+p_2)$

(c) use either x_1 or x_2. And also use either x_3 or x_4

But need amount of x_1 (or x_2) $= x_3$ (or x_4)

Thus $V = \frac{y}{\min(p_1,p_2) + \min(p_3,p_4)} \quad \Rightarrow \quad e = u_0\left[\min(p_1,p_2) + \min(p_3,p_4)\right]$

(d) $x_1^H = u_0 \quad x_2^H = u_0^2 \quad \Rightarrow \quad u_0 = x_1 = \sqrt{x_2}$

Since $x_1 = \sqrt{x_2}$, utility is $u(x_1, x_2) = \min(x_1, \sqrt{x_2})$