

FOR PRACTICE ONLY (2014)

Economics 610
Advanced Microeconomic Theory

Fall 2013
Professor Schlesinger

Exam 1

Answer all 5 questions below. Be sure to explain how you derive your answers.

1. State whether each of the following statement is TRUE or FALSE and provide your reasons for saying so. All of the credit for this question is based upon your explanations.

- (a) $e(v(y, p_1, p_2), p_1, p_2) = y$, where e denotes the expenditure function and v denotes the indirect utility function.
- (b) $x_1^M(y, p_1, p_2) = x_1^H(v(y, p_1, p_2), p_1, p_2)$.
- (c) Lexicographic preferences satisfy the axiom of local nonsatiation.
- (d) For a country exporting green eggs and importing ham (both normal goods), an increase in the price of green eggs will cause the country to export more green eggs.

Note: *A good graphical explanation will suffice for this answer.*

2. A competitive market consists of 15 potential firms. Firm i , $i = 1, 2, \dots, 15$, must pay a fixed fee of $2i$ to open its doors. If firm i does not produce, it does need to pay this “quasi-fixed cost.” The cost function for firm i is $c(q) = q^2 + 2i$ if $q > 0$ and $c(0) = 0$. [For example, firm 15 faces the cost function $c(q) = q^2 + 30$ for any positive q , but with $c(0) = 0$.] The competitive market price of this product is currently $p = 5$.

- (a) Will all 15 firms decide to “open their doors?” How much will each of the 15 firms produce in the short run?
- (b) How would your answer to part (a) change if the market price fell to $p = 1$?

3. Mrs. Donizetti likes to eat, which of course requires that she have food. She would prefer not to work and live a life of leisure, but works enough to buy food at a price of p per unit of food. The weekly rent on her apartment is paid for by her ex-husband, but she must buy all of her own food every week. She has 80 hours of time each week when she is not sleeping, which she divides between working at an hourly rate of w and “leisure” (which consists of sitting in front of her stereo with a bucket of food listening to opera). In addition, the government imposes a weekly income tax on all wages so that Mrs. Donizetti pays tw to the government for every hour that she works, where $t > 0$. Both food and leisure are normal goods.

- (a) Write down Mrs. Donizetti’s weekly budget constraint and explain it.
- (b) Suppose that there is a small increase in the tax rate t . Will Mrs. Donizetti decide to work more or work less? Explain carefully. (*To receive full credit, you must show your solution analytically. If you cannot do this, I will give you partial credit for a well-explained graphical answer.*)

4. Two firms have identical cost functions $c(q_i) = \frac{1}{4}q_i^2$. The market demand is given by $q^D = 20 - p$.
- Find the Cournot-Nash equilibrium price in this market.
 - Find the price in a Stackelberg equilibrium, with firm 1 as the leader.
5. Find the indirect utility function [either $v(y, p_1, p_2)$ or $v(y, p_1, p_2, p_3, p_4)$] for each of the following direct utility functions.
- $u(x_1, x_2) = \min(x_1, x_2)$
 - $u(x_1, x_2) = [\min(x_1, x_2)]^2$
 - $u(x_1, x_2) = \max(x_1, x_2)$
 - $u(x_1, x_2, x_3, x_4) = \min(x_1, x_2) + \min(x_3, x_4)$

Exam 2

Answer all five (5) questions below. Be sure to explain how you derive your answers.

- A firm has the cost function $c(y, w_1, w_2) = y^2 \sqrt{w_1 w_2}$.
 - Use Shephard's Lemma to find the conditional factor demand functions $x_1(y, w_1, w_2)$ and $x_2(y, w_1, w_2)$.
 - Find the production function $f(x_1, x_2)$.
- Consider the production function $f(x_1, x_2) = x_1^{1/2} + x_2^{1/2}$.
 - Find the elasticity of substitution σ for this technology.
 - Find the factor demands $x_1(p, w_1, w_2)$ and $x_2(p, w_1, w_2)$.
 - Find the profit function $\pi(p, w_1, w_2)$.
- Rich buys two goods every day, x_1 and x_2 . On Monday the prices were on sale with $(p_1, p_2) = (2, 4)$. On Tuesday the prices were $(p_1, p_2) = (6, 3)$. On one of these two days, Rich bought bundle $A = (1, 2)$ while on the other day he bought bundle $B = (2, 1)$. If Rich is a rational guy, with preferences satisfying the weak axiom of revealed preference (WARP), on which day did Rich buy bundle A? Explain. [All credit for this problem is based on your explanation.]

4. (a) Consider the utility function $u(w) = \frac{1}{b-1}(a+bw)^{1-(1/b)}$, where $b > 0$, $b \neq 1$, and we assume that $a+bw > 0$. (Note that a might be negative, but if it is, w is restricted so that $a+bw > 0$.) Determine whether relative risk aversion is increasing or decreasing in wealth (or neither).

(b) Suppose that a consumer's risk-averse utility exhibits constant absolute risk aversion (CARA). The consumer has $\$W$ in starting cash plus a lottery ticket \tilde{X} , where \tilde{X} pays a prize between $\$0$ and $\$1000$. Let π denote Pratt's risk premium for this consumer's risky wealth. Show that the risk premium π will be the same for every possible value of starting cash $\$W$.

5. The firm 1099-Products Inc. produces in two plants, located in Akron and Boise respectively. Plant A (Akron) has the production function $y_A = f_A(x_1, x_2) = \min(x_1, x_2)^{1/2}$ and Plant B (Boise) has the production function $y_B = f_B(x_1, x_2) = 2 \min(x_1, x_2)^{1/2}$. Inputs have the same costs (w_1 and w_2) at both locations.

(a) Find the cost functions for each plant: $c_A(y_A, w_1, w_2)$ and $c_B(y_B, w_1, w_2)$. [Not surprisingly, you should be able to verify that it is cheaper for 1099-Products to produce output in Boise than in Akron, given a choice of one or the other.]

(b) Of course, the firm will produce most efficiently by utilizing both plants. It will produce any specific quantity y by choosing the cheapest combination of y_A and y_B such that $y_A + y_B = y$. Find the cost function for the firm $c(y, w_1, w_2)$ and verify that is cheaper than producing in either plant alone.