For Practice Only

The University of Alabama

Finance 601 | **Spring 2014**
Finance Theory I | **Professor Schlesinger**

Exam 1

Answer all 5 questions below. You must show how you derive your answers to receive full credit on each question.

1. Let $\tilde{X}$ be a binomial random variable with $n = 2$ and $p = 1/2$. Let $\tilde{Y}$ also be binomial, but with $n = 3$ and $p = 1/3$. Finally, let $\tilde{Z}$ also be random with two equally likely outcomes of $z = 0.5$ and $z = 1.5$.

   (a) Rank these three random variables by first-order stochastic dominance, or explain why they cannot be ranked.

   (b) Rank these three random variables by second-order stochastic dominance, or explain why they cannot be ranked.

2. A risk averse investor has quadratic utility $u(y) = y - ky^2$, where we assume that $y < \frac{1}{2k}$. There is also a zero-mean background risk $\tilde{\varepsilon}$, which is independent of any distribution of wealth. Define the derived utility function as $v(y) \equiv Eu(y + \tilde{\varepsilon})$. Show that $v(y)$ is an affine transformation of $u(y)$.

3. Consider a two-period dynamic-choice investment decision. The investor is assumed to have preferences represented by the linex utility function, $u(y) = y - e^{-\gamma y}$, where $\gamma > 0$. We also assume that the risk-free rate of interest is zero. Will this investor behave myopically? In other words, will the investor make the same decisions as if he/she were only investing for one period?

4. An individual with initial wealth w_0 and decreasing absolute risk aversion (DARA) invests in a risk-free asset and one risky asset, with gross returns R_f and $\tilde{R}$ respectively, where $E\tilde{R} > R_f$. The individual currently invests \$*a* in the risky asset. Suppose that there is a small increase in the risk-free rate R_f. We still have $E\tilde{R} > R_f$. Will this individual invest more or less than \$*a* in the risky asset? You should show your result analytically and explain it in economic terms.

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5. Suppose Mr. Crusoe maximizes utility of consumption over T+1 dates. Let intertemporal utility be given by

$$U(c_0, c_1, c_2, \ldots, c_T) \equiv u(c_0) + \frac{u(c_1)}{1+\delta} + \frac{u(c_2)}{(1+\delta)^2} + \ldots + \frac{u(c_T)}{(1+\delta)^T},$$

where $u(c)$ is increasing and concave, and $\delta > 0$. Mr. Crusoe has a fixed initial wealth of w_0 and he can save at a fixed rate of interest $r_f > \delta$. Of course, he cannot borrow, since he receives no income after date t = 0. Also, he cannot consume less than zero in any period.

(a) Show that Mr. Crusoe's consumption will increase in every period. (Note: Be sure to carefully specify Mr. Crusoe's optimization problem.)

(b) Suppose that $u(c) = -e^{-\gamma c}$, with $\gamma > 0$. Show that Mr. Crusoe's consumption will increase by a constant dollar amount from period to period.

(c) Suppose instead that $u(c) = c$, so that utility is not strictly concave, and suppose that there are only two dates, i.e. T = 1. Find the optimal consumption stream, c_0 and c_1.

Exam 2

Answer all 5 questions below.

1. Consider a principal-agent model in which the principal owns the contingent claim (x_1, x_2), where $x_1 > x_2$. The probability of state 1 depends upon agent effort e, with $p(e)$ a continuous, increasing and concave function. Effort by the agent costs c units of utility per unit of effort. The principal and agent are both risk averse. The agent will work for the principal for the contingent fee of (a_1, a_2), so long as the agent receives his reservation utility u_o. If the agent's level of effort e is observable, the payment (a_1, a_2) will only be made if the contracted level of effort was provided.

(a) Suppose that effort is observable. Show that the optimal (first-best) contract requires the agent to provide more effort than he desires. [That is, for the given contingent payment of (a_1, a_2), the agent would prefer to provide less effort than specified in the contract.]

(b) If effort is unobservable, does the optimal (second-best) contract (a_1^*, a_2^*) entail first-best Pareto-efficient risk sharing? Explain carefully.

2. Consider the two state adverse selection model of Rothschild and Stiglitz (1976) and assume that there are an equal number of good risks and bad risks in society. The probabilities of a loss satisfy $0 < p_G < p_B < 1$.

(a) What is the price for a pooling contract in terms of p_G and p_B?

(b) Show why a pooling equilibrium cannot exist in this model.

(c) How does your answer to (b) change if we use Wilson's (1977) model of equilibrium?

3. Mr. Cameron receives an income the beginning of each of two periods. His income at date $t = 0$ is y_0. At date $t = 1$, his income is random $\tilde{y}_1$, with $E\tilde{y}_1 = y_0$. Both borrowing and lending are possible at a risk-free rate of interest r_f. Mr. Cameron maximizes his intertemporal utility of consumption: $u(y_0 - s) + \frac{1}{1+\delta} Eu(\tilde{y}_1 + s(1 + r_f))$, where $r_f = \delta$.

(a) Will Mr. Cameron borrow or save money (or neither) at date $t = 0$?

(b) Suppose that Mr. Cameron's income at date $t = 1$ was $\tilde{y}_2$ instead of $\tilde{y}_1$, where $\tilde{y}_2$ is riskier than $\tilde{y}_1$ in the sense of Rothschild & Stiglitz (1970). Would he save more than in your answer to part (a)?

4. Consider a dynamic, two-period model of investment. The individual has an initial wealth of \$1 and preferences are give by the utility function $u(w) = \ln w$. We assume that the individual does not discount her utility of future consumption ($\delta = 0$) and that the gross risk-free interest rate is $R_f = 1.20$. The individual must decide how much to invest in the risky and risk-free assets at dates $t = 0$ and $t = 1$.

Let $\tilde{R}_t$ denote the random gross returns for the risky asset at date t. The distributions are i.i.d. at each point in time, with $\tilde{R}_t$ taking on the value 1.5 or 1.0, each with an equally likely chance. Let z denote the investor's wealth at the end of the first period. The individual then invests $\alpha_1 z$ in the risky asset at date $t = 1$.

(a) Find the optimum investment in stock at date $t = 1$, $\alpha_1^* z$.

(b) Determine the value function (i.e. the Bellman function) $v(z)$ for this dynamic programming problem.

5. Consider a complete-market exchange economy with N identical risk-averse consumers and S states of nature. Each individual i is endowed with the contingent wealth claim $\omega_i(z_k) = \frac{1}{N} z_k$, in state k, where z_k indicates the aggregate wealth in state k. In other words, the endowments are also identical. For simplicity, assume that $R_f = 1$.

(a) Show that not trading contingent claims (i.e., each individual consumes his or her own endowment) is Pareto efficient.
[Note: this corresponds to the weighting vector for individual utilities $\vec{\lambda} = (\frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N})$]

(b) Show that the state price densities defined by $\pi(z_k) = \dfrac{u'(\omega(z_k))}{\sum_{s=1}^{S} p_s u'(\omega(z_s))}$ will lead to an equilibrium in which there is no trading.

Hint for part (b): Recall that for a competitive equilibrium, we have one of the Pareto-efficient allocations, with $\lambda_i = 1/\xi_i$, where ξ_i is the Lagrange multiplier from individual i's optimal consumption program. Without losing generality, this assumes that $\sum_1^N (1/\xi_i) = 1$. Write out the first order conditions for a competitive equilibrium in the above case, where there is no trading in equilibrium. Also, since $R_f = 1$, what do you know about $\sum_{s=1}^{S} p_s \pi(z_s)$?