Insurance Management Professor Schlesinger

Problems

1. Consider distribution functions with supports contained in (*a*,*b*). Define the following preferences functionals:

(i) $V(F) \equiv \mu_F$ (the mean of F)

(ii) $V(F) \equiv$ the median of F

(iii) $V(F) \equiv$ max Supp (F)

(iv) $V(F) \equiv$ min Supp (F)

(v) $V(F) \equiv f(\mu_F, \sigma_F^2)$ (function of mean and variance)

For preference functionals (i)-(iv) above, find examples of simple lotteries to convince yourself that these preferences are unrealistic. Do you think preference functional (v) is realistic? Why or why not?

2. Lottery *A* yields either 0 or 10 unites of utility, each with an equal probability. Lottery *B* yields 5 units of utility and no risk. Which lottery is preferred by a risk averter and why?

3. Assume $u''' > 0$ and define absolute prudence as $p(y) \equiv \dfrac{-u'''(y)}{u''(y)}$.

Define $v(y) \equiv -u'(y)$ and note that $v(y)$ has the properties of a risk-averse utility function. Show the equivalence of:

(a) *u* exhibits DARA

(b) $p(y) > r(y) \forall y$

(c) $v(y)$ is more risk averse than $u(y)$.

4. A consumer has \$100 in wealth plus a lottery ticket. The lottery ticket pays a prize of \$100 with probability p = ½. Otherwise, the prize is zero. The consumer has von Neumann-Morgenstern utility exhibiting constant relative risk aversion, with the level of relative risk aversion $y = 1$.

(a) What is the lowest price that consumer would accept to sell this lottery ticket?

(b) If the consumer had \$100, but did not own the lottery ticket, how much would she be willing to pay to buy the ticket?

5. Suppose that $F \sim Unif[a,b]$ and that $G \sim Unif[A,B]$.
Give conditions on {*a*,*b*,*A*,*B*} such that F FSD G. Give conditions such that F SSD G.

6. Let $\tilde{X}$ denote an equally weighed lottery with prizes 5 and 10. Let $\tilde{Y}$ denote an equally weighted lottery with prizes 0 and 15. Explain how $\tilde{Y}$ is a simple mean-preserving spread of $\tilde{X}$. Find $\tilde{\varepsilon}$ such that $E(\tilde{\varepsilon} | x) = 0 \ \forall x$ and $\tilde{Y} =_d \tilde{X} + \tilde{\varepsilon}$.

7. Both $\tilde{x}$ and $\tilde{y}$ have supports contained in (a,b). Show that $E\tilde{x} = E\tilde{y}$ if only if $\int_a^b F(x)dx = \int_a^b G(x)dx.$

8. Consider a two-state loss model, where a loss of size *L* occurs with probability *p*, $L \leq W$. Let $v(y)$ be a more risk-averse utility than $u(y)$. Show the following:

(a) If the premium loading $\lambda = 0$, then $\alpha_u^* = \alpha_v^* = 1$

(b) If $\lambda > 0$, then $\alpha_u^* < \alpha_v^* < 1$ [Hint: Use Pratt's Theorem]

9. Let $\tilde{Y} = W - \tilde{x}$, with insurance available for the loss $\tilde{x}$ and with $\lambda > 0$. Let α^* denote the optimal coinsurance level.

(a) Suppose that preferences satisfy CARA. Show that α^* remains unchanged for $\tilde{Y} = kw - \tilde{x}, k > 0$.

(b) Suppose that preferences satisfy CRRA. Show that α^* remains unchanged for $\tilde{Y} = k\tilde{w} - k\tilde{x}, k > 0.$

10. Consider a state-claims model with two states of nature. What is the slope of the price line for insurance contracts? Use this information to demonstrate Mossin's Theorem.

11. Let $u(y) \equiv y - ky^2, k > 0, y < 1/2k$. Let $\tilde{\varepsilon}$ be an independent background risk, $E\tilde{\varepsilon} = 0$. Show that the optimal level of insurance α^* is the same both with and without the background risk. (Note that $u''' = 0$ for quadratic utility.)

12. Consider a model of insurance demand with a "fair" price and a possibility of insurer default. Explain why an increase in risk aversion might <u>not</u> lead to an increase in the optimal level of insurance.

13. Consider a model of deductible insurance. Show that Mossin's Theorem also holds for deductibles:

(1) $\lambda = 0 \Rightarrow$ Full insurance is optimal, $D^* = 0.$

(2) $\lambda > 0 \Rightarrow$ Partial coverage is optimal, $D^* > 0$.

14. Set up a model of insurance coverage for a policy with an upper limit. The indemnity payment is specified is $I(x) = \min(x, \theta)$ where θ is the upper limit, chosen by the consumer. Assume the premium is set as $P(\theta) = (1+\lambda)E[I(\tilde{x})]$. What is the first-order condition for the optimal choice of an upper limit?

15. Let $\tilde{x}$ be a random variable that has a payoff of $-p_2$ with probability p_1, and a payoff of $+p_1$ with probability p_2. Note that $E\tilde{x}=0$. Consider $t\tilde{x}$, $t>0$ and define $k(t)$ via $\mu \sim \mu + t\tilde{x} + k(t)$. Suppose that risk aversion is of order one, so that $\lim_{t \to 0^+} k'(t) > 0$. Show that the slope of an indifference curve in state-claims space is steeper than p_1 / p_2, even as t approaches zero.

16. Yaari's "Dual Theory" models a preference functional for choice under risk as:

$$V(F) \equiv \int y d[g(F(y))],$$

where F is the cumulative distribution function and $g:[0,1] \to [0,1]$ is an increasing function, and g is strictly concave if there is risk aversion. Consider a two-state world with probability p_i for state $i=1,2$. What do the indifference curves look like in state-claims space? Is risk aversion of order one or of order two?

17. *MFÖ* Insurance Company has reinsurance with International Extreme Reinsurance (IERe). *MFÖ* has issued an insurance policy covering losses up to € 6.000.000. Suppose there is a € 4.000.000 claim that is paid on this policy. Explain how much of this claim (how many euros) will be reimbursed by IERe for each of the following reinsurance arrangements. Be sure to explain your answers.

(a) *MFÖ* has an excess-loss reinsurance, with a stop-loss limit set at € 3.000.000.

(b) *MFÖ* has a surplus share reinsurance for 3 lines, with *MFÖ* retention limit equal to € 2.000.000.

(c) *MFÖ* has a surplus share reinsurance for 3 lines, with *MFÖ* retention limit equal to € 1.000.000.

(d) *MFÖ* has a surplus share reinsurance for 3 lines, with *MFÖ* retention limit equal to € 5.000.000.

18. Consider the Rothschild-Stiglitz adverse-selection model, but with 3 types of insureds: good, medium, and bad, where $p_G < p_M < p_B$. Characterize the R-S equilibrium. Be sure to consider pooling contracts, separating contracts and <u>mixed</u> contracts (i.e. where two types pool with a separate contract for the third type.)

19. Consider a two-state model of insurance demand. The insured has two possible levels of effort. Suppose insurance is offered, but at a price that includes a premium loading $\lambda > 0$. The loading λ can be thought of as a competitive loading to cover marketing expenses. Characterize insurance prices and insurance demand under moral hazard.