Matrikel-Nr.:__________________________

UNIVERSITÄT KONSTANZ

Insurance Management

Juli 2013 Professor Schlesinger

KLAUSUR

Before starting: write your Matrikel Number, not your name, above (upper right).

*Answer **any 4** of the following 5 questions. Answers may be written in German or in English. Each question answered is worth 25 points (100 points in total). To get full credit you must show how you derive your answers. You need to earn at least 50 points to pass the course. Note that I will not grade all 5 questions, so only answer 4.* **Please write neatly.**

YOU MUST TURN IN THIS QUESTION SHEET WITH YOUR EXAM!

Use the following abbreviated mortality table to answer question 1.

Age	Probability of Dying	Number of People	Number of Deaths
70	0.03951	627,416	24,789
71	0.04330	602,627	26,094
72	0.04765	576,533	27,472
73	0.05264	549,061	28,903
74	0.05819	520,159	30,268

1. Volker Wagener just turned 71 years old and wishes to purchase a three-year term life insurance policy. This policy pays € 200,000 if death occurs during the three year term of the policy. Any premium is paid at the beginning of the year, and death benefits are paid out at the end of the year of death. Assume that the interest rate for borrowing and lending is 5%. Further assume that there is a premium loading of 10% (i.e., $\lambda = 0.1$).

(a) Determine the single premium that he would be charged for such a policy.

(b) Determine the level premium that he would be charged for such a policy.

2. Pohlmeier Insurance, AG, has a surplus-share reinsurance treaty with Breyer Reinsurance. Pohlmeier Insurance has a retention limit of € 300,000 per policy. Breyer Reinsurance has agreed to take up to 2 lines of Reinsurance. Each Pohlmeier policy pays for full coverage, up to the policy limit.

(a) How much would Pohlmeier Insurance pay and how much would Breyer Reinsurance pay for a loss of € 100,000 on each of the following insurance policies?

(i) One with a € 80,000 policy limit?
(ii) One with a € 750,000 policy limit?
(iii) One with a € 1,000,000 policy limit?

(b) Suppose that the loss distribution for policy (ii) above has a uniform density on [0, 750,000], i.e. $f(x) = 1/(750{,}000)\ \forall x$. Find the fair premium for the surplus-share reinsurance for policy (ii) above.

(c) Surplus-share reinsurance is an example of proportional reinsurance. Often, insurers will arrange for non-proportional types of reinsurance, such as excess-loss reinsurance. Suppose that instead of a surplus-share treaty, Breyer Reinsurance offered an excess-of-loss reinsurance treaty, with a stop-loss limit of € 300,000 <u>for each insurance policy</u> written by Pohlmeier Insurance. How would your answers to (i) – (iii) in part (a) change?

3. Consider the adverse-selection model of Rothschild & Stiglitz.

    (a) Explain the circumstances under which a separating equilibrium might not exist in their model.

    (b) In a situation such as you described in part (a), suppose the government provides a pooling contract in which everyone receives full insurance at a fair pooling price. No private insurance is allowed, but individuals are free to decline the purchase of the government insurance. Explain whether such a pooling contract might succeed.

4. Consider preferences represented by Yaari's dual theory, with the preference functional defined as

$$V(F) \equiv \int y d[g(F(y))].$$ (Note: this is NOT expected utility preferences)

Here $F(y)$ is the true cumulative distribution function for wealth y and $g(F(y)) = (F(y))^{1/2}$ is a probability distortion. A consumer with these preferences has an initial wealth w_0 and faces a loss of size L. The probability of a loss occurring is $p = 0.25$.

    (a) Draw an indifference curve for this individual in state-claims space. Indicate her marginal rate of substitution.

    (b) Will Mossin's Theorem hold for this individual? Explain carefully. (Note that a graphical explanation will suffice.)

5. . Consider an individual purchasing a coinsurance policy. In addition to an insurable loss $\tilde{x}$, there is a zero-mean background risk $\tilde{\varepsilon}_1$ that is statistically independent from the insurable loss. The premium contains a proportional premium loading factor $\lambda > 0$. Preferences are represented by the so-called linex utilty:

$$u(y) = by - e^{-\gamma y}, \text{ where } b \geq 0 \text{ and } \gamma > 0.$$

Note that for $b = 0$, we have CARA (i.e. exponential utility). For the case where $b = 0$, we denote the optimal coinsurance level as α_0.

    (a) Show that the level of coinsurance purchased is a decreasing function of b.

    (b) Now suppose that the background risk becomes riskier in the sense of Rothschild and Stiglitz (1970). In particular, suppose that $\tilde{\varepsilon}_2$ MPIR $\tilde{\varepsilon}_1$. Show that if $b = 0$ and we replace $\tilde{\varepsilon}_1$ with $\tilde{\varepsilon}_2$, then α_0 will not change.

    (c) Suppose that $b > 0$ and we replace $\tilde{\varepsilon}_1$ with $\tilde{\varepsilon}_2$. Show that the optimal level of insurance is higher with $\tilde{\varepsilon}_2$. [Hint: Define $f(\varepsilon) \equiv e^{-\gamma\varepsilon}$. Is $f(\varepsilon)$ increasing or decreasing? Is it concave or convex? Now consider $Ef(\tilde{\varepsilon})$. What does Rothschild and Stiglitz (1970) tell us about $Ef(\tilde{\varepsilon}_2)$ versus $Ef(\tilde{\varepsilon}_1)$? How does this relate to the first-order condition?]

<u>Checklist before turning in your exam:</u>

-- WRITE YOUR MATRIKEL-NUMBER ON THE FIRST PAGE OF THIS QUESTION SHEET

-- CHECK TO BE SURE THAT YOU ANSWERED <u>4</u> (AND ONLY 4) QUESTIONS

-- DO NOT FORGET TO INCLUDE ALL OF THE PAGES WITH YOUR ANSWERS

-- REMEMBER TO TURN IN THIS QUESTION SHEET WITH YOUR EXAM