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The Theory of Insurance Demand

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Abstract: This chapter presents the basic theoretical model of insurance demand in a one-period expected utility setting. Models of coinsurance and of deductible insurance are examined along with their comparative statics with respect to changes in wealth, prices and attitudes towards risk. The single risk model is then extended to account for multiple risks, such as insolvency risk and background risk. It is shown how only a subset of the basic results of the single-risk model is robust enough to extend to models with multiple risks.

The theory of insurance demand is often regarded as the purest example of economic behavior under uncertainty. Interestingly, whereas twenty years ago most upper-level textbooks on microeconomics barely touched on the topic of uncertainty, much less insurance demand, textbooks today at all levels often devote substantial space to the topic. The purpose of this chapter is to present the basic model of insurance demand that imbeds itself not only into the other papers in this volume and in the insurance literature, but also in many other settings within the finance and economics literatures. Since models that deal with non-expected utility analysis are dealt with elsewhere in this Handbook, I focus only on the expected-utility framework.

Many models look at markets for trading risk, but typically such risks are designed for trades. Insurance, on the other hand, deals with a personal risk. In treating such a risk, the consumer can try to modify the risk itself through methods such as prevention, which is another topic in this book. Alternatively, the consumer could try to pool risks with a large group of other consumers, but organizing such a group would pose some problems of its own. We can view insurance as an intermediary that in a certain sense organizes such risk pooling. Such an approach is generically referred to as "risk financing."

The device offered by the insurer is one in which, for a fixed premium, the insurer promises an indemnity for incurred losses. Of course, there are many variations on this theme, as one can see from gleaning the pages of this Handbook. From a purely theoretical viewpoint, the model presented in section 1 of this chapter should be viewed as a base model, from which all other models deviate.

In some ways, insurance is simply a financial asset. However, whereas most financial assets are readily tradable and have a risk that relates to the marketplace, insurance is a contract contingent on an individual's own personal wealth changes. This personal nature of insurance is what distinguishes it from other financial assets. It also exacerbates problems of informational asymmetry, such as moral hazard and adverse selection, which also are dealt with elsewhere in this Handbook.

The preponderance of insurance models isolate the insurance-purchasing decision. The consumer decides how much insurance to buy for a well-defined risk. And indeed, this chapter starts out the same way in section 1. However, when multiple risks face the consumer, it is not likely to be optimal to decide how to handle each risk separately.

Rather, some type of overall risk-management strategy is called for. Simultaneous decisions over multiple risk-management instruments is beyond the scope of this chapter, However, even if we make an insurance decision in isolation, the presence of these other risks is most likely going to affect our choice. The second part of this chapter shows how the presence of other risks – so-called "background risk" – impacts the consumer's insurance-purchasing decision.

1. The Singe Risk Model

Insurance contracts themselves can be quite complicated, but the basic idea is fairly simple. For a fixed premium P the insurer will pay the insured a contingent amount of money that depends upon the value of a well-defined loss. This insurance payment is referred to as the *indemnity*.[1]

To make the model concrete, consider an individual with initial wealth $W > 0$. Let the random variable $\tilde{x}$ denote the amount of the loss, where we assume that $0 \leq x \leq W$ to avoid bankruptcy issues. The insurance indemnity is contingent only on x and will be written as $I(x)$. We often assume that $I(x)$ is non-decreasing in x and that $0 \leq I(x) \leq x$, though neither of these assumptions is necessary to develop a theory of insurance demand. We do, however, assume that the realization of $\tilde{x}$ is costlessly observable by all parties and that both parties agree on the distribution of the random

[1] Technically an "indemnity" reimburses an individual for out-of-pocket losses. I will use this terminology to represent generically any payment from the insurance company. For some types of losses, most notably life insurance, the payment is not actually indemnifying out-of-pocket losses, but rather is a specified fixed payment.

variable $\tilde{x}$. Models that do not make these last two assumptions are dealt with elsewhere in this Handbook.

The insurer, for our purpose, can be considered as a risk-neutral firm that charges a market-determined price for its product. The individual is considered to be risk averse with von Neumann-Morgenstern utility of final wealth given by the function $u(\cdot)$, where u is assumed to be everywhere twice differentiable with $u' > 0$ and $u'' < 0$. The assumption of differentiability is not innocuous. It is tantamount in our model to assuming that risk aversion is everywhere of order 2.[2]

1.1 Proportional Coinsurance

The simplest type of indemnity payment is one in which the insurer pays a fixed proportion, say α, of the loss. Thus, $I(x) = \alpha x$. This type of insurance indemnity is often referred to as *coinsurance*, since the individual retains (or "coinsures") a fraction $1-\alpha$ of the loss. If $\alpha = 1$, the insurer pays an indemnity equal to the full value of the loss and the individual is said to have *full insurance.*

An assumption that $0 \le I(x) \le x$ here is equivalent to assuming that $0 \le \alpha \le 1$. The case where $\alpha > 1$ is often referred to as *over insurance*. The case where $\alpha < 0$ is referred to by some as "selling insurance," but this description is incorrect. If $\alpha < 0$, the individual is taking a short position in his or her *own* loss; whereas selling insurance is taking a short position in someone else's loss.

To consider the insurance-purchasing decision, we need to specify the insurance premium as a function of the indemnity. The most general form of the premium is

(1) $P\left[I(\cdot)\right] = E\left[I(\tilde{x}) + c[I(\tilde{x})]\right].$

Here E denotes the expectation operator and $c(\cdot)$ is a cost function, where $c[I(x)]$ denotes the cost of paying indemnity $I(x)$, including any market-based charges for assuming the risk $I(\tilde{x})$. Note that P itself is a so-called *functional*, since it depends upon the function $I(\cdot)$.

As a base case, we often consider $c\left[I(x)\right] = 0 \ \forall x$. This case is often referred to as the case of "*perfect competition*" in the insurance market, since it implies that insurers receive an expected profit of zero, and the premium is referred to as a *fair premium*.[3]

The premium, as defined in (1), is a bit too general to suit our purpose here. See Gollier (2012) for more discussion of this general premium form. We consider here the simplest case of (1) in which the expected cost is proportional to the expected indemnity; in particular

(2) $P(\alpha) = E(\alpha\tilde{x} + \lambda\alpha\tilde{x}) = \alpha(1+\lambda)E\tilde{x}$,

where λ is called the *loading factor*, $\lambda \geq 0$. Thus, for example, if λ equals 0.10, the insurer would charge a premium equal to the expected indemnity plus an additional 10 percent to cover the insurer's expenses and profit margin. The consumer's final wealth can then be expressed as a random variable, dependent upon the choice of the level of coverage α,

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[2] See Segal and Spivak (1990). Although extensions to the case where *u* is not everywhere differentiable are not difficult, they are not examined here. See Schlesinger (1997) for some basic results.

[3] Obviously real-world costs include more than just the indemnity itself, plus even competitive insurers earn a "normal return" on their risk. Thus, we do not really expect $c\left[I(x)\right] = 0$. That the zero-profit case is labeled "perfect competition" is likely due to the seminal paper by Rothschild and Stiglitz (1976). We also note, however, that real-world markets allow for the insurer to invest premium income, which is omitted here. Thus, zero-costs might not be a bad approximation for our purpose of developing a simple model. The terminology "fair premium" is taken from the game-theory literature, since such a premium in return for the random payoff $I(\tilde{x})$ represents a "fair bet" for the insurer.

(3) $\tilde{Y}(\alpha) \equiv W - \alpha(1+\lambda)E\tilde{x} - \tilde{x} + \alpha\tilde{x}$.

The individual's objective is choose α so as to maximize his or her expected utility

(4) $$\underset{\alpha}{\text{maximize}} \quad E[u(\tilde{Y}(\alpha))],$$

where we might or might not wish to impose the constraint that $0 \le \alpha \le 1$.

Solving (4) is relatively straightforward, yielding a first-order condition for the unconstrained objective

(5) $$\frac{dEu}{d\alpha} = E[u'(\tilde{Y}(\alpha)) \cdot (\tilde{x} - (1+\lambda)E\tilde{x})] = 0 .$$

The second-order condition for a maximum holds trivially from our assumption that $u'' < 0$. Indeed, $d^2Eu / d\alpha^2$ is negative <u>everywhere</u>, indicating that any α^* satisfying (5) will be a global maximum. The fact that $E[u(\tilde{Y}(\alpha))]$ is globally concave in α also turns out to be quite important in later examining various comparative statics.

Evaluating $dEu / d\alpha$ at $\alpha = 1$ shows that

(6) $$\left.\frac{dEu}{da}\right|_{\alpha=1} = Eu'(W - \alpha(1+\lambda)E\tilde{x})(\tilde{x} - (1+\lambda)E\tilde{x}) = -\lambda(E\tilde{x})u'(W - \alpha(1+\lambda)E\tilde{x}).$$

Since $u' > 0$, the sign of (6) will be zero if $\lambda = 0$ and will be negative if $\lambda > 0$. Together with the concavity of $Eu(\tilde{Y}(\alpha))$ in α, this implies the following result, usually referred to as *Mossin's Theorem*: [4]

[4] The result is often attributed to Mossin (1968), with a similar analysis also appearing in Smith (1968).

Mossin's Theorem: *If proportional insurance is available at a fair price* $(\lambda = 0)$*, then full coverage* $(\alpha^* = 1)$ *is optimal. If the price of insurance includes a positive premium loading* $(\lambda > 0)$*, then partial insurance* $(\alpha^* < 1)$ *is optimal.*

Note that Mossin's Theorem does not preclude a possibility that $\alpha^* < 0$ in the unconstrained case. Indeed, evaluating $dEu/d\alpha$ at $\alpha = 0$ when $\lambda > 0$, yields

(7) $$\frac{dEu}{d\alpha}\Big|_{\alpha=0} = -\lambda Eu'(\tilde{Y}(0)) \cdot E\tilde{x} + Cov(u'(\tilde{Y}(0), \tilde{x}).$$

Since the covariance term in (7) is positive and does not depend on λ, we note that there will exist a unique value of λ such that the derivative in (7) equals zero. At this value of λ, zero coverage is optimal, $\alpha^* = 0$. For higher values of λ, $\alpha^* < 0$. Since $Eu(\tilde{Y}(\alpha))$ is concave in α, $\alpha = 0$ will be a constrained optimum whenever the unconstrained optimum is negative. In other words, if the price of insurance is too high, the individual will not purchase any insurance.

As long as the premium loading is non-negative, $\lambda \geq 0$, the optimal level of insurance will be no more than full coverage, $\alpha^* \leq 1$. If, however, we allow for a negative premium loading, $\lambda < 0$, such as might be the case when the government subsidizes a particular insurance market, then over insurance, $\alpha^* > 1$, will indeed be optimal in the case where α is unconstrained. Strict concavity of $Eu(\tilde{Y}(\alpha))$ in α once again implies that full insurance, $\alpha = 1$, will be a constrained optimum for this case, when over insurance is not allowed.

It may be instructive for some readers to compare the above results with the so-called *portfolio problem* in financial economics. The standard portfolio problem has an investor allocate her wealth between a risky and a riskless asset. If we let *A* denote final wealth when all funds are invested in a riskless asset, and let $\tilde{z}$ denote the random excess payoff above the payoff on the riskless asset, the individual must choose a weight β, such that final wealth is

(8) $Y(\beta) = (1-\beta)A + \beta(A+\tilde{z}) = A + \beta\tilde{z}$.

A basic result in the portfolio problem is that $\operatorname{sgn}\beta^* = \operatorname{sgn} E\tilde{z}$. If we set $A \equiv W - (1+\lambda)E\tilde{x}$, $\tilde{z} \equiv (1+\lambda)E\tilde{x} - \tilde{x}$, and $\beta = (1-\alpha)$, then (8) is equivalent to (3). Noting that sgn $E\tilde{z} = \operatorname{sgn}\lambda$ in this setting, our basic portfolio result is exactly equivalent to Mossin's Theorem. Using equation (8), we can think of the individual starting from a position of full insurance $(\beta = 0)$ and then deciding upon the optimal level to coinsure, β^*. If $\lambda > 0$, then coinsurance has a positive expected return, so that any risk averter would choose to accept some of the risk $\beta^* > 0$ (i.e. $\alpha^* < 1$).

1.2 Effects of Changes in Wealth and Price

Except in the special case of a binary risk, it is often difficult to define what is meant by the *price* and the *quantity* of insurance. Since the indemnity is a function of a random variable and since the premium is a functional of this indemnity function, both price and quantity – the two fundamental building blocks of economic theory – have no direct counterparts for insurance. However, for the case of coinsurance, we have the level of coinsurance α and the premium loading factor λ, which fill in nicely as proxy measures of quantity and price respectively.

If the individual's initial wealth changes, but the loss exposure remains the same, will more or less insurance be purchased? In other words, is insurance a "normal" or an "inferior" good? Clearly, if $\lambda = 0$, then Mossin's Theorem implies that full insurance remains optimal. So let us consider the case where $\lambda > 0$, but assume that λ is not too large, so that $0 < \alpha^* < 1$. Since $Eu(\tilde{Y}(\alpha))$ is concave in α, we can determine the effect of a higher *W* by differentiating the first-order condition (5) with respect to *W*. Before doing this however, let us recall a few items from the theory of risk aversion.

If the Arrow-Pratt measure of local risk aversion, $r(y) = -u''(y)/u'(y)$, is decreasing in wealth level *y*, then preferences are said to exhibit decreasing absolute risk aversion (DARA). Similarly, we can define constant absolute risk aversion (CARA) and increasing absolute risk aversion (IARA). We are now ready to state the following result.

Proposition 1: *Let the insurance loading* λ *be positive. Then for an increase in the initial wealth level W,*

(i) *the optimal insurance level* α^* *will decrease under DARA,*

(ii) *the optimal insurance level* α^* *will be invariant under CARA,*

(iii) *the optimal insurance level* α^* *will increase under IARA.*

Proof: Let *F* denote the distribution of $\tilde{x}$. By assumption, the support of *F* lies in the interval [0,*W*]. Define $x_0 \equiv (1+\lambda)E\tilde{x}$. Assume DARA. Then we note that $r(y_1) < r(y_0) < r(y_2)$ for any $y_1 > y_0 > y_2$, and, in particular for $y_0 = W - \alpha^*(1+\lambda)E\tilde{x} - x_0 + \alpha x_0$. Differentiating (5) with respect to *W*, we obtain

$$(9) \qquad \left.\frac{\partial^2 Eu}{\partial\alpha\partial W}\right|_{\alpha^*} = \int_0^W u''(Y(\alpha^*))(x- \ (1+\lambda)E\tilde{x})dF$$

$$= -\int_0^{x_0} r(Y(\alpha^*))u'(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF$$

$$-\int_{x_0}^{W} r(Y(\alpha^*))u'(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF$$

$$< -r(y_0)\left[\int_0^W u'(Y(\alpha^*))(x-(1+\lambda)E\tilde{x})dF\right] = 0.$$

Thus increasing wealth causes α^* to fall.

The cases where preferences exhibit CARA or IARA can be proved in a similar manner. ■

We should caution the reader that DARA, CARA and IARA do not partition the set of risk-averse preferences. Indeed each of these conditions is shown to be sufficient for the comparative-static effects in Proposition 1, though none is necessary.[5]

The case of CARA is often used as a base case, since such preferences eliminate any income effect. However, a more common and, by most standards, realistic assumption is DARA, which implies that insurance is an inferior good. One must use caution in using this terminology however. It is valid only for the case of a fixed loss exposure $\tilde{x}$. Since real-world loss exposures typically increase as wealth increases, we do not necessarily expect to see richer individuals spending less on their insurance purchases, *ceteris paribus*.[6] We do, however, expect that they would spend less on the same loss exposure.

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[5] If we wish to strengthen the claims in Proposition 1 to hold for every possible starting wealth level and every possible random loss distribution, then DARA, CARA and IARA would also be necessary.

[6] If the support of $\tilde{x}$ is [0, L], it may be useful to define $W \equiv W_0 + L$. If the loss exposure is unchanged, an increase in W can be viewed as an increase in W_0. More realistically, an increase in W will consist of increases in both W_0 and L.

In a similar manner, we can examine the effect of an increase in the loading factor λ on the optimal level of insurance coverage. Differentiating the first-order condition (5) with respect to λ obtains

(10) $$\left.\frac{\partial^2 Eu}{\partial\alpha\partial\lambda}\right|_{\alpha^*} = -[E\tilde{x}Eu'(\tilde{Y}(\alpha^*))] - \alpha E\tilde{x}\frac{\partial^2 Eu}{\partial\alpha\partial W}\ .$$

The first term on the right-hand side of equation (10) captures the substitution effect of an increase in λ. This effect is negative due to the higher price of insurance. A higher λ implies that other goods (not insurance) are now relatively cheaper, so that the individual should save some of the premium and use it to buy other items. The second term on the right-hand side of (10) captures an income effect, since a higher premium would lower overall wealth, *ceteris paribus*. For a positive level of α, which we are assuming, this effect will be the opposite sign of $\partial^2 Eu/\partial\alpha\partial W$. For example, under DARA, this income effect is positive: the price increase lowers the average wealth of the individual, rendering him or her more risk averse. This higher level of risk aversion, as we shall soon see, implies that the individual will purchase more insurance. If this second (positive) effect outweighs the negative substitution effect, insurance can be considered a Giffen good.[7] More comprehensively, the following result is a direct consequence of equation (10) and Proposition 1.

Proposition 2: *Let the insurance loading be positive, with* $0 < \alpha^* < 1$. *Then, insurance cannot be a Giffen good if preferences exhibit CARA or IARA, but may be Giffen if preferences exhibit DARA.*

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[7] A necessary and sufficient condition for insurance not to be Giffen is given by Briys, Dionne and Eeckhoudt (1989).

1.3 Changes in Risk and in Risk Aversion

If the loss distribution *F* changes, it is sometimes possible to predict the change in optimal insurance coverage α^*. Conditions on changes to *F* that are both necessary and sufficient for α^* to increase are not trivial, but can be found by applying a Theorem of Gollier (1995) to the portfolio problem, and then using the equivalence of the portfolio problem and the insurance problem. Although this condition is very complex, there are several sufficient conditions for α^* to rise due to a change in risk that are relatively straightforward. Since this topic is dealt with elsewhere in this Handbook (Eeckhoudt and Gollier, 2012), I do not detour to discuss it any further here.

A change in risk aversion, on the other hand, has a well-defined effect upon the choice of insurance coverage. First of all, we note that for an insurance premium that is fair, $\lambda = 0$, any risk-averse individual will choose an insurance policy with full coverage, $\alpha^* = 1$. If, however, the insurance premium includes a positive premium loading, $\lambda > 0$, then an increase in risk aversion will always increase the level of insurance. More formally,

Proposition 3: *Let the insurance loading be positive, with* $0 < \alpha^* < 1$. *An increase in the individual's degree of risk aversion at all levels of wealth will lead to an increase in the optimal level of coverage, ceteris paribus.*

Proof: Let α^*_u denote the optimal level of coverage under the original utility function *u*. Let *v* denote a uniformly more risk-averse utility function. We know from Pratt (1964), that there exists a function $g:\mathbb{R}\to\mathbb{R}$ such that $v(y)=g[u(y)]$, where $g'>0$ and $g''<0$.

Since *v* is a risk-averse utility function, we note that $Ev(\tilde{Y}(\alpha))$ is concave in α. Thus, consider the following:

(11) $$\left.\frac{dEv}{d\alpha}\right|_{\alpha_u^*}=\left.\frac{dEg[u]}{d\alpha}\right|_{\alpha_u^*}=\int_0^W g'[u(Y(\alpha_u^*))]u'(Y(\alpha_u^*))(x-(1+\lambda)E\tilde{x})dF$$

$$>g'[u(y_0)]\left\{\int_0^{x_0}u'(Y(\alpha_u^*))(x-(1+\lambda)E\tilde{x})dF+\int_{x_0}^{W}u'(Y(\alpha_u^*))(x-(1+\lambda)E\tilde{x})dF\right\}=0$$

where x_0 and y_0 are as defined in the proof of Proposition 1, and where the inequality follows from the concavity of *g*. This last expression equals zero by the first-order condition for α_{u}^*.

Since $Ev(\tilde{Y}(\alpha))$ is concave in α, the inequality in (11) implies that $\alpha_{\text{v}}^*>\alpha_{\text{u}}^*$. ■

1.4 Deductible Insurance

Although proportional coinsurance is the simplest case of insurance demand to model, real-world insurance contracts often include fixed co-payments per loss called "deductibles." Indeed, optimal contracts include deductibles under fairly broad assumptions. Under fairly simple but realistic pricing assumptions, straight deductible policies can be shown to be optimal.[8] In this section, we examine a few aspects of insurance demand when insurance is of the deductible type.

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[8] See the essay by Gollier (2012) in this Handbook for a detailed analysis of the optimality of deductibles.

For deductible insurance, the indemnity is set equal to the excess of the loss over some predetermined level. Let *L* denote the supremum of the support of the loss distribution, so that *L* denotes the maximum possible loss. By assumption, we have $L \le W$. Define the deductible level $D \in [0, L]$ such that $I(x) \equiv \max \ (0, x - D)$. If *D*=0, the individual once again has full coverage, whereas *D*=*L* now represents zero coverage. One complication that arises, is that the general premium, as given by equation (1), can no longer be written as a function of only the mean of the loss distribution, as in (2). Also, it is difficult to find a standard proxy for the *quantity* of insurance in the case of deductibles.[9]

In order to keep the model from becoming overly complex, we assume here that the distribution *F* is continuous, with density function *f*, so that $dF(x) = f(x)dx$. We will once again assume that the insurance costs are proportional to the expected indemnity, so that the premium for deductible level *D* is given by

$$(12) \qquad P(D) = (1+\lambda)E[I(\tilde{x})] = (1+\lambda)\int_D^L (x-D)dF(x)$$

$$= (1+\lambda)\int_D^L [1-F(x)]dx,$$

where the last equality is obtained via integration by parts.

Using Leibniz Rule, one can calculate the marginal premium reduction for increasing the deductible level,[10]

$$(13) \qquad P'(D) = -(1+\lambda)(1-F(D)).$$

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[9] Meyer and Ormiston (1999) make a strong case for using $E[I(\tilde{x})]$, although its often much simpler to use *D* as an inverse proxy for insurance demand.

[10] Leibniz rule states that

$$\frac{d}{dt}\int_{a(t)}^{b(t)} H(x,t)dx = H(b,t)b'(t) - H(a,t)a'(t) + \int_{a(t)}^{b(t)} \frac{\partial H}{\partial t}dx.$$

By increasing the deductible level, say by an amount ΔD, the individual receives a lower payout in all states of the world for which the loss exceeds the deductible. The likelihood of these states is $1-F(D)$. While it is true that the likelihood will also change as *D* changes, this effect is of secondary importance and, due to our assumption of a continuous loss distribution, disappears in the limit.

Following the choice of a deductible level *D* and using the premium as specified in (12), final wealth can be written as

(14) $\tilde{Y}(D) = W - P(D) - \min(\tilde{x}, D)$.

The individual's objective is now to choose the best deductible *D* to

(15) maximize $E[u(\tilde{Y}(D))]$, where $0 \leq \mathrm{D} \leq \mathrm{L}$.

Assume that the premium loading is non-negative, $\lambda \geq 0$, but not so large that we obtain zero coverage as a corner solution, $D^* = L$. The first-order condition for the maximization in (15), again using Leibniz rule, is

(16)
$$\frac{dEu}{dD} = -P'\int_0^D u'(W - \mathrm{P} - x)dF + (-P'-1)\int_D^L u'(W-P-D)dF$$

$$= -P'\int_0^D u'(W-P-x)dF + (-P'-1)(1-F(D))u'(W-P-D) = 0.$$

The first term in either of the center expressions in (16) represents the marginal net utility benefit of premium savings from increasing *D*, conditional on the loss not exceeding the deductible level. The second term is minus the net marginal utility cost of a higher deductible, given that the loss exceeds the deductible. Thus, (16) has a standard economic interpretation of choosing *D** such that marginal benefit equals marginal cost.

The second-order condition for the maximization in (16) can be shown to hold as follows.

$$\text{(17)} \qquad \frac{d^2Eu}{dD^2} = (1+\lambda)(-f(D))\int_0^D u'(W-P-x)dF + (-P')u'(W-P-D)f(D)$$

$$+(-P')^2\int_0^D u''(W-P-x)dF + (1+\lambda)(-f(D))(1-F(D))u'(W-P-D)$$

$$+(-P'-1)(-f(D))u'(W-P-D) + (-P'-1)^2(1-F(D))''(W-P-D).$$

Multiplying all terms containing *f*(*D*) in (17) above by $(1-F(D))/(1-F(D))$ and simplifying, yields

$$\text{(18)} \qquad \frac{d^2Eu}{dD^2} = \frac{-f(D)}{1-F(D)}\left[-P'\int_0^D u'(W-P-x)dF + (-P'-1)(1-F(D))u'W-P-D)\right]$$

$$+\left[(-P')^2\int_0^D u''(W-P-x)dF + (-P'-1)^2(1-F(D))u''(W-P-D)\right] < 0$$

The first term in (18) is zero by the first-order condition, while the second term is negative from the concavity of *u*, thus yielding the inequality as stated in (18).

To see that Mossin's Theorem can be extended to the case of deductibles, rewrite the derivative in (16) as

$$\text{(19)} \qquad \frac{dEu}{dD} = (1-F(D))\left[(1+\lambda)\int_0^L u'(W-P-\min(x,D))dF - u'(W-P-D)\right].$$

If $\lambda = 0$, then (19) will be negative for any $D > 0$, and is easily seen to equal zero when $D = 0$. For $\lambda > 0$, (19) will be positive at $D = 0$, so that the deductible should be increased. Therefore, Mossin's Theorem also holds for a choice of deductible. It also is straightforward to extend the comparative-static results of Propositions 1-3 to the case of deductibles as well, although we do not provide the details here.

Another type of insurance indemnity is for so-called "upper-limit insurance." Under this type of insurance, the insurer pays for full coverage, but only up to some pre-specified limit θ. For losses above this limit the indemnity is simply $I(x) = \theta$. Unlike a deductible policy, which requires the individual to bear small losses on his or her own, an upper-limit policy requires the individual to bear the cost of losses over size θ on his or her own. Whereas deductible insurance is the most preferred indemnity structure for a risk averter, as shown in this handbook by Gollier (2012) using stochastic dominance arguments, it turns out that upper-limit policies are the least preferred type of indemnity structure.[11]

Mossin's Theorem also can be extended to the case of upper-limit insurance policies as well, although the mathematical details are a bit messy and are not presented here.[12]

2. The Model with Multiple Risks

Although much is to be learned from the basic single-risk model, rarely is the insurance decision made with no other uncertainty in the background. This so-called background risk might be exogenous or endogenous. In the latter case decisions on how to best handle risk cannot usually be decided in isolation on a risk-by-risk basis. Rather,

[11] Gollier (2012) uses the stochastic-dominance methodology proposed by Gollier and Schlesinger (1996) to show the optimality of deductibles for a risk averter.. A similar type of argument can be constructed to show that upper-limit policies are least preferred.

[12] The problematic issue deals with differentiability of the objective function. The details can be found in Schlesinger (2006).

some type of comprehensive risk management policy must be applied.[13] However, even in the case where the background risk is exogenous and independent of the insurable risk, we will see that the mere presence of background risk affects the individual's insurance choice.

The existence of uninsurable background risk is often considered a consequence of incomplete markets for risk sharing. For example, some types of catastrophic risk might contain too substantial an element of non-diversifiable risk, including a risk of incorrectly estimating the parameters of the loss distribution, to be insurable. Likewise, non-marketable assets, such as one's own human capital, might not find ready markets for sharing the risk. Similarly, problems with asymmetry of information between the insurer and the insured, such as moral hazard and/or adverse selection, might preclude the existence of insurance markets for certain risks.

We begin the next section by examining a type of secondary risk that is always present for an insurable risk, but almost universally ignored in insurance theory; namely the risk that the insurer does not pay the promised indemnity following a covered loss. The most obvious reason for non-payment is that the insurer may be insolvent and not financially capable of paying its claims in full. However, other scenarios are possible. For instance, there might be some events that void insurance coverage, such as a probationary period for certain perils, or exclusion of coverage in situations of civil unrest or war.[14] Even if the insurer pays the loss in full, it may decide to randomly investigate a claim thereby substantially delaying payment. In such an instance, the delay

[13] This question was first addressed by Mayers and Smith (1983) and Doherty and Schlesinger (1983a). The special case of default risk was developed by Doherty and Schlesinger (1990).

[14] Although not modeled in this manner, the possibility of a probationary period is examined by Eeckhoudt, et al. (1988), who endogenize the length of probation.

reduces the present value of the indemnity, which has the same effect as paying something less than the promised indemnity.

2.1 The Model with Default Risk

We consider here an insurance model in which the insurer might not pay its claims in full. To keep the model simple, we consider only the case of a full default on an insured's claim in which a loss of a fixed size either occurs or does not occur. Let the support of the loss distribution be $\{0, L\}$, where a loss of size L occurs with probability *p*, $0 < p < 1$. Let α once again denote the share of the loss paid as an indemnity by the insurer, but we now assume that there is only a probability *q*, $0 < q < 1$, that insurer can pay its claim, and that with probability $1-q$ the claim goes unpaid.[15] As a base case, we consider a fair premium, which we calculate taking the default risk into account as $P(\alpha) = \alpha pqL$.

Obviously such a premium is not realistic, since for $q < 1$ it implies that the insurer will default almost surely. More realistically the insurance will contain a premium loading of $\lambda > 0$. Thus $P(\alpha) = \alpha p[(1+\lambda)q]L$. Since *P*, α, *p* and *L* are known or observable, the consumer observes only $q(1+\lambda)$, rather than *q* and λ separately. It is the consumer's *perception* of *q* and λ that will cause a deviation in insurance purchasing from the no-default-risk case. Since we only concern ourselves with how default risk affects insurance demand, the base case of a "fair premium" with $\lambda = 0$ seems like a good place to start.

[15] In a two-state (loss vs. no loss) model, there is no distinction between coinsurance and deductibles. A coinsurance rate α is identical to a deductible level of $D = (1-\alpha)L$.

Given our model, states of the world can be partitioned into three disjoint sets: states in which no loss occurs, states in which a loss occurs and the insurer pays its promised indemnity, and states in which a loss occurs but the insurer pays no indemnity. We assume that the individual's loss distribution is independent of the insurer's insolvency. Thus, the individual's objective can be written as

(20) $$\underset{\alpha}{\text{maximize}} \quad Eu = (1-p)u(Y_1) + pqu(Y_2) + p(1-q)u(Y_3)$$

where

$$Y_1 \equiv W - \alpha pqL$$

$$Y_2 \equiv W - \alpha pqL - L + \alpha L$$

$$Y_3 \equiv W - \alpha pqL - L$$

The first-order condition for maximizing (20) is

(21) $$\frac{dEu}{d\alpha} = -(1-p)pqLu'(Y_1) + pq(1-pq)Lu'(Y_2) - p(1-q)pqLu'(Y_3) = 0.$$

If we evaluate the derivative in (21) when $\alpha = 1$ and $q < 1$, we have $Y_1 = Y_2 > Y_3$ so that

(22) $$\left.\frac{dEu}{d\alpha}\right|_{\alpha=1} = p^2qL(1-q)[u'(Y_1) - u'(Y_3)] < 0 .$$

Given the concavity of $u(\cdot)$, equation (22) implies that $\alpha^* < 1$ and clearly Mossin's Theorem does not hold in the presence of default risk. Consequently, we have

(23) $$Y_1 > Y_2 > Y_3 .$$

In the presence of default risk, although we can purchase “nominally full insurance,” with $\alpha^* = 1$, this does not fully insure the individual, since the insurer might not be able to pay a valid claim. Indeed, in the case where the insurer does not pay a filed claim, the individual is actually worse off than with no insurance, since the individual also loses his or her premium. The higher the level of insurance, the higher the potential loss of premium. Thus, it is not surprising that $\alpha^* = 1$ is not optimal.[16]

It also is not difficult to show that, in contrast to the case with no default risk, an increase in risk aversion will not necessarily lead to an increase in the level of insurance coverage. Although a more risk-averse individual would value the additional insurance coverage absent any default risk, higher risk aversion also makes the individual fear the worst-case outcome (a loss and an insolvent insurer) even more. More formally, let $v(\cdot)$ be a more risk-averse utility function than $u(\cdot)$. As in section 1.3, we know there exists an increasing concave function *g*, such that $v(y) = g[u(y)]$ for all *y*.

Without losing generality, assume that $g'[u(Y_2)] = 1$, so that $g'[u(Y_1)] < 1 < g'[u(Y_3)]$. Now, we can calculate the following:

$$\left.\frac{dEv}{d\alpha}\right|_{\alpha^*} = -g'[u(Y_1)](1-p)pqLu'(Y_1) + pq(1-pq)Lu'(Y_2) - g'[u(Y_3)]p(1-q)pqLu'(Y_3). \tag{24}$$

Comparing (24) with (21), we see that one of the negative terms on the right-hand side in (24) is increased in absolute magnitude while the other is reduced. However, it is not possible to predetermine which of these two changes will dominate, *a priori*. Thus, we cannot predict whether α^* will increase or decrease.

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[16] Note that if there is no default risk with $q = 1$, then $u'(Y_1) = u'(Y_2)$ implying that $\alpha^* = 1$, as we already know from Mossin's Theorem. Also, if insurance in default pays for most of the claim (as opposed to none of the claim), it is possible for full coverage or even more-than-full coverage to be optimal. See Doherty and Schlesinger (1990) and Mahul and Wright (2007).

Using similar arguments, it is easy to show that insurance is not necessarily an inferior good under DARA, as was the case without default risk. A somewhat more surprising result is that, under actuarially fair pricing, an increase in the probability of insolvency does not necessarily lead to a higher level of coverage. To see this, use the concavity of $Eu(Y(\alpha))$ in α, which is easy to check, and calculate

(25) $$\left.\frac{\partial^2 Eu}{\partial\alpha\partial q}\right|_{\alpha^*} = p\alpha L[H(\alpha^*)] + p^2 qL[u'(Y_3) - u'(Y_2)],$$

where $H(\alpha)$ is defined as the derivative in the first-order condition (21), with $u(Y)$ replaced by the utility function $-u'(Y)$. The level of insurance coverage will increase, due to an increase in *q*, if and only if (25) is positive. Although the second term on the right-hand side of (25) is positive, the first term can be either positive or negative. For example, if *u* exhibits DARA, it is straightforward to show that $-u'$ is a more risk averse utility than *u*. Therefore, by our results on increases in risk aversion, $H(\alpha^*)$ might be either positive or negative.

There are two, and only two, circumstances in which the form of the utility function u will yield $d\alpha^*/dq > 0$, regardless of the other parameters of the model (assuming fair prices). The first is where *u* is quadratic, so that $H(\alpha) = 0$ for all α. The second is where *u* satisfies CARA, and which case $-u'$ and *u* represent the same risk-averse preferences.[17] Hence, $H(\alpha^*) = 0$. We also know for any risk-averse utility *u*, that $d\alpha^*/dq > 0$ for q sufficiently close to $q = 1$. This follows since $\alpha^* = 1$ for $q = 1$, but $\alpha^* < 1$ for $q < 1$.

[17] This is easiest to see by noting that $-u'$ is an affine transformation of *u*.

2.2 An Independent Background Risk

As opposed to a default risk, we now suppose that the insurer pays all of its claims, but that the individual's uninsured wealth prospect is $W + \tilde{\varepsilon} - \tilde{x}$, where $\tilde{x}$ once again represents the insurable loss and where $\tilde{\varepsilon}$ represents a zero-mean background risk that is independent of $\tilde{x}$. We assume that the support of the distribution of $\tilde{\varepsilon}$ is not the singleton $\{0\}$ and that $W + \tilde{\varepsilon} - \tilde{x} > 0$ almost surely. It is assumed that $\tilde{\varepsilon}$ cannot be insured directly. We wish to examine the effect of $\tilde{\varepsilon}$ on the choice of insurance level α^*.

The case of an independent background risk is easily handled by introducing the so-called *derived utility function* which we define as follows:

(26) $$v(y) \equiv Eu(y + \tilde{\varepsilon}) = \int_{-\infty}^{\infty} u(y + \varepsilon) dG(\varepsilon),$$

where $G(\cdot)$ is the distribution function for $\tilde{\varepsilon}$. The signs of the derivatives of *v* are easily seen to be identical to those of *u*. Note that we can now write

(27) $$\max_{\alpha} Eu(\tilde{Y}(\alpha) + \tilde{\varepsilon}) = \int_0^L \int_{-\infty}^{\infty} u(Y(\alpha) + \varepsilon) dG(\varepsilon) dF(x) \equiv \int_0^L v(Y(\alpha)) dF(x)$$

$$= Ev(\tilde{Y}(\alpha)).$$

In other words, $v(Y(\alpha))$ is simply the "inner part" of the iterated integral in (27). Finding the optimal insurance level for utility *u* in the presence of background risk $\tilde{\varepsilon}$, is identical to finding the optimal insurance level for utility *v*, absent any background risk.

For example, suppose *u* exhibits CARA or that *u* is quadratic. Then it is easy to show in each case that *v* is an affine transformation of *u*, so that background risk has no effect on the optimal choice of insurance.[18]

More generally, we know from Proposition 3 that more insurance will be purchased whenever the derived utility function $v(\cdot)$ is more risk averse than $u(\cdot)$. A sufficient condition for this to hold is *standard risk aversion* as defined by Kimball (1993). A utility function exhibits standard risk aversion "if every risk that has a negative interaction with a small reduction in wealth also has a negative interaction with any undesirable, independent risk." [Kimball (1993) p. 589] Here "negative interaction" means that risk magnifies the reduction in expected utility. Kimball shows that standard risk aversion is characterized by decreasing absolute risk aversion and decreasing absolute prudence, where absolute risk aversion is $r(y) = -u''(y)/u'(y)$ and absolute prudence is $\eta(y) = -u'''(y)/u''(y)$.

It is easy to show that DARA is equivalent to $\eta(y) > r(y)\ \forall y$. Since DARA implies prudence (i.e. $u'''(y) > 0$), then under DARA the function $-u'(y)$ represents a risk-averse utility of its own. The condition $\eta(y) > r(y)$ thus implies that $-u'(\cdot)$ is a more risk-averse utility than $u(\cdot)$. Similarly, it follows that decreasing absolute prudence or "DAP" implies that $u''''(y) < 0$ and that $u''(\cdot)$ is a more risk-averse utility function than $-u'(\cdot)$.

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[18] For CARA, $v(y) = ku(y)$ and for quadratic utility $v(y) = u(y) + c$, where $k = E[\exp(r\tilde{\varepsilon})] > 0$, *r* denotes the level of risk aversion and $c = -t\,\mathrm{var}\left(\tilde{\varepsilon}\right)$ for some $t > 0$. Gollier and Schlesinger (2003) show that these are the only two forms of utility for which *v* represents preferences identical to *u*.

Let $\pi(\text{y})$ denote the risk premium, as defined by Pratt (1964), for utility $u(\cdot)$, given base wealth y and fixed zero-mean risk $\tilde{\varepsilon}$. Similarly, let $\pi_1(\text{y})$ and $\pi_2(\text{y})$ denote the corresponding risk premia for utilities $-u'(\cdot)$ and $u''(\cdot)$ respectively. That is,

$$\begin{aligned} Eu(y+\tilde{\varepsilon}) &\equiv u(y-\pi(y)) \\ -Eu'(y+\tilde{\varepsilon}) &\equiv -u'(y-\pi_1(y)) \\ Eu''(y+\tilde{\varepsilon}) &\equiv u''(y-\pi_2(y)). \end{aligned} \tag{28}$$

Standard risk aversion thus implies that $\pi_2(y) > \pi_1(y) > \pi(y) > 0 \quad \forall y$. Thus, we have the following set of inequalities

$$-\frac{v''(y)}{v'(y)} = \frac{-Eu''(y+\tilde{\varepsilon})}{Eu'(y+\tilde{\varepsilon})} = \frac{-u''(y-\pi_2)}{u'(y-\pi_1)} > \frac{-u''(y-\pi_1)}{u'(y-\pi_1)} > \frac{-u''(y)}{u'(y)} \ . \tag{29}$$

The first inequality follows from DAP while the second inequality follows from DARA. Consequently $v(\cdot)$ is more risk-averse than $u(\cdot)$.[19]

The above result taken together with our previous result on increases in risk aversion, implies the following:

Proposition 4:

(a) *If insurance has a zero premium loading,* $\lambda = 0$, *then full coverage is optimal in the presence of an independent background risk.*

(b) *If insurance premia include a positive loading,* $\lambda > 0$, *then partial coverage is optimal in the presence of an independent background risk.*

(c) *If insurance premia include a positive loading,* $\lambda > 0$, *and utility exhibits standard*

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[19] Another simple proof that standard risk aversion is sufficient for the derived utility function to be more risk averse appears in Eeckhoudt and Kimball (1992). Standard risk aversion is stronger than necessary, however. See Gollier and Pratt (1996).

risk aversion, then more coverage is purchased in the presence of an independent zero-mean background risk.

Remark: Parts (a) and (b) above do not require $E\tilde{\varepsilon}=0$. They are direct applications of Mossin's Theorem to utility $v(\cdot)$. Although the discussion above is for proportional coinsurance, part (c) of Proposition 4 also applies to deductibles, since it only relies upon $v(\cdot)$ being more risk-averse than $u(\cdot)$.

2.3 Non-independent Background Risk

Obviously the background risk need not always be statistically independent of the loss distribution. For example, if $\tilde{\varepsilon}=\tilde{x}$ then final wealth is risk free without insurance.[20] Buying insurance on $\tilde{x}$ would only introduce risk into the individual's final wealth prospect. Consequently, zero coverage is optimal, even at a fair price, $\lambda=0$. For example, suppose the individual's employer provides full insurance coverage against loss $\tilde{x}$. We can represent this protection by $\tilde{\varepsilon}$ as described here; and thus no further insurance coverage would be purchased.

Similarly, if $\tilde{\varepsilon}=-\tilde{x}$ then final wealth can be written as $\tilde{Y}=W-2\tilde{x}$ with no insurance. Treating $2\tilde{x}$ as the loss variable, Mossin's Theorem implies that full insurance on $2\tilde{x}$ will be optimal at a fair price. This can be achieved by purchasing insurance with a coinsurance level of $\alpha^*=2$. Although this is nominally "200% coverage," it is defacto merely full coverage of $2\tilde{x}$. If insurance is constrained to exclude over-insurance, then $\alpha=1$ will be the constrained optimum. For insurance markets with a premium loading $\lambda > 0$, Mossin's Theorem implies that $\alpha^* < 2$. In this case, a constraint of no over-insurance might or might not be binding.

For more general cases of non-independent background risk, it becomes difficult to predict the effects on insurance purchasing. Part of the problem is that there is no general measure of dependency that will lead to unambiguous effects on insurance demand. Correlation is not sufficient since other aspects of the distributions of $\tilde{x}$ and $\tilde{\varepsilon}$, such as higher moments, also are important in consumer choice.[21] Alternatives measures of dependence, many based on stochastic dominance, do not lead to definitive qualitative effects on the level of insurance demand.

For example, suppose we define the random variable $\tilde{\varepsilon}'$ to have the same marginal distribution as $\tilde{\varepsilon}$, but with $\tilde{\varepsilon}'$ statistically independent of $\tilde{x}$. We can define a partial stochastic ordering for $W + \tilde{\varepsilon} - \tilde{x}$ versus $W + \tilde{\varepsilon}' - \tilde{x}$. If, for example, we use second-degree stochastic dominance, we will be able to say whether or not the risk-averse consumer is better off or worse off with $\tilde{\varepsilon}$ or $\tilde{\varepsilon}'$ as the source of background risk; but we will not be able to say whether the level of insurance demanded will be higher or lower in the presence of background risk $\tilde{\varepsilon}$ versus background risk $\tilde{\varepsilon}'$.

Some research has used more sophisticated partial orderings to examine the behavior of insurance demand in the presence of a background risk that is not statistically independent from the loss distribution. For the most part, this work has focused on comparing insurance demands both with and without the background risk. Aboudi and Thon (1995) do an excellent and thorough job of characterizing many of the potential

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[20] In stating that two random variables are equal, we mean that they each yield the same value in every state of nature, not simply that they have equal distributions.

partial orderings, albeit in a discrete probability space, but they only whet our appetite for applying these orderings to insurance demand. Hong et al (2011) also characterize some of these orderings and they show that one of these orderings in particular, namely positive [or negative] expectation dependence, is both necessary and sufficient to claim a variant of Mossin's Theorem for coinsurance:[22]

Generalized Mossin's Theorem: *In the presence of a background risk* $\tilde{\varepsilon}$, *less than* [*more than*] *full coverage is always demanded by a risk averter at a fair price if and only if losses are positively* [*negatively*] *expectation dependent on* $\tilde{\varepsilon}$

Other types of dependencies are of course possible. Eeckhoudt and Kimball (1992), for example, use one particular partial ordering, assuming that the conditional distribution of $\tilde{\varepsilon}$ given x_1 dominates the conditional distribution of $\tilde{\varepsilon}$ given x_2 via third-degree stochastic dominance, for every $x_1 < x_2$. One example of such a relationship would be that the conditional distributions of $\tilde{\varepsilon}$ all have the same mean and variance, but the conditional distributions of $\tilde{\varepsilon}$ become more negatively skewed as losses increase. Eeckhoudt and Kimball go on to show that such a negative dependency between $\tilde{\varepsilon}$ and $\tilde{x}$ leads to an increase in insurance demand in the presence of background risk, whenever

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[21] Doherty and Schlesinger (1983b) use correlation, but restrict the joint distribution of $\tilde{x}$ and $\tilde{\varepsilon}$ to be bivariate normal. For other joint distributions, correlation is not sufficient. A good discussion of this insufficiency can be found in Hong et al (2008).

[22] Losses $\tilde{x}$ are positively expectation dependent on $\tilde{\varepsilon}$ if $E(\tilde{x} \mid \tilde{\varepsilon} \le k) \le E(\tilde{x})\ \ \forall k$. In a certain sense, a smaller value of $\tilde{\varepsilon}$ implies that expected losses will be smaller. Negative expectation dependence simply reverses the second inequality in the definition. It should be noted that Hong et al (2011) do not consider the case of a positive premium loading. Thus, their theorem only extends one part of Mossin's Theorem. See also the paper by Dana and Scarsini (2007), which uses similar dependence structures to examine the optimal contractual form of insurance

preferences exhibit standard risk aversion. Important to note here, is that even with the strong third-degree stochastic dominance assumption, risk aversion alone is not strong enough to yield deterministic comparative statics.

In an interesting paper, Tibiletti (1995) compares the demand for insurance for a change in background risk from $\tilde{\varepsilon}'$ to $\tilde{\varepsilon}$, where $\tilde{\varepsilon}'$ is statistically independent from $\tilde{x}$ and has the same marginal distribution as $\tilde{\varepsilon}$. She uses the concept of concordance as her partial ordering. In particular, if $H(\varepsilon, x)$ is the joint distribution of the random vector $(\tilde{\varepsilon}, \tilde{x})$ and $G(\varepsilon, x)$ the distribution of $(\tilde{\varepsilon}', \tilde{x})$, then *H* is *less concordant* than *G* if $H(\varepsilon, x) \geq G(\varepsilon, x) \;\; \forall \varepsilon, x$. In other words, *G* dominates *H* by joint first-degree stochastic dominance.[23]

However, even using concordance, we need to make fairly restrictive assumptions on preferences to yield deterministic comparisons between optimal levels of insurance purchases. In particular, suppose that we restrict the degree of relative prudence, $y\eta(y) = -yu'''(y) / u''(y)$, to be no greater than one. Then for *H* less concordant than *G*, more insurance will be purchased under *H*; i.e., more insurance is purchased in the presence of background risk $\tilde{\varepsilon}$ than in the presence of the independent background risk $\tilde{\varepsilon}'$. While this result seems intuitively appealing, just as the result of Eeckhoudt and

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[23] To the best of my knowledge, Tibiletti (1995) also introduces the use of *copulas* into insurance models. Copulas allow one to describe the joint distribution of $(\tilde{\varepsilon}, \tilde{x})$ as a joint distribution function of the marginal distributions of $\tilde{\varepsilon}$ and $\tilde{x}$, which is a type of normalization procedure. This allows one to both simplify and generalize the relationship between *H* and *G*. The use of particular functional forms for the *copula* allows one to parameterize the degree of statistical association between $\tilde{x}$ and $\tilde{\varepsilon}$. See Frees and Valdez (1998) for a survey of the use of *copulas*.

Kimball (1992), neither follows automatically if we assume only risk aversion for consumer preferences.[24]

As a final case consider a background risk that changes size as the state of nature changes. In particular, consider a model with two loss states, but with a possibly different zero-mean background risk in each potential state. The consumer chooses α to maximize

(30) $$Eu = (1-p)Eu(W + \tilde{\varepsilon}_1 - \alpha(1+\lambda)pL) + pEu(W + \tilde{\varepsilon}_2 - \alpha(1+\lambda)pL - L + \alpha L) .$$

The first-order condition for maximizing (30) can be written as

(31) $$-c_1 Eu'(W + \tilde{\varepsilon}_1 - \alpha(1+\lambda)pL) + c_2 Eu'(W + \tilde{\varepsilon}_2 - \alpha(1+\lambda)pL - L + \alpha L) = 0 ,$$

where c_1 and c_2 are positive constants. The first term (negative) is the marginal utility cost of higher coverage if no loss occurs, which stems from the higher premium. The second term (positive) is the marginal utility benefit of the higher indemnity if a loss occurs.

If the consumer is prudent, $u''' > 0$, the presence of $\tilde{\varepsilon}_i$ will increase the marginal utility in both states.[25] If marginal utility is increased by the same proportion in each state, then simultaneously eliminating both $\tilde{\varepsilon}_i$ will not have any effect on the optimal insurance demand. The same coinsurance level α would be optimal both with and without the background risks.

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[24] The fact that detrimental changes in the background risk $\tilde{\varepsilon}$ do not necessarily lead to higher insurance purchases under simple risk aversion is examined by Eeckhoudt, Gollier and Schlesinger (1996), for the case where the deterioration can be measured by first- or second-degree stochastic dominance. Keenan, Rudow and Snow (2008) extend the analysis to consider deteriorations via background risks that either reduce expected utility or increase expected marginal utility.

[25] This follows easily using Jensen's inequality, since marginal utility is convex under prudence.

On the other hand, if we only eliminate $\tilde{\varepsilon}_1$, then there would be background risk only in the loss state. In this case, the marginal utility cost of insurance would fall and more insurance would be purchased due to prudence. In other words, insurance would be higher if there was only a background risk in the loss state. The reason for this is a precautionary motive. Although the risk $\tilde{\varepsilon}_2$ cannot be hedged, having more wealth in the loss state makes this $\tilde{\varepsilon}_2$-risk more bearable under prudence. Thus, the consumer has a precautionary motive to buy more insurance. If we eliminated only the $\tilde{\varepsilon}_2$-risk, but kept the $\tilde{\varepsilon}_1$-risk, the same precautionary motive would be used to reduce the demand for insurance in order to save some of the premium dollars in the no-loss state.[26]

3. Concluding Remarks

Mossin's Theorem is often considered to be the cornerstone result of modern insurance economics. Indeed this result depends only on risk aversion for smooth preferences, such as those found in the expected-utility model.[27] On the other hand, many results depend on stronger assumptions than risk aversion alone, and research has turned in this direction. Stronger measures of risk aversion, such as those of Ross (1981) and of Kimball (1993), have helped in our understanding more about the insurance-purchasing decision.

One common "complaint," that I hear quite often from other academics, is that these restrictions on preferences beyond risk aversion are too limiting. These critics

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[26] The intuition behind this precautionary effect can be found in Eeckhoudt and Schlesinger (2006). Further results on how such differential background risk can affect insurance decisions can be found in Fei and Schlesinger (2008).

[27] Actually, this result depends on the differentiability of the von Neumann-Morgenstern utility function. "Kinks" in the utility function can lead to violations of Mossin's result. See, for example, Eeckhoudt, Gollier and Schlesinger (1997).

might be correct, if our goal is to guess at reasonable preferences and then see what theory predicts. However, insurance demand is not just a theory. I doubt there is anyone reading this that does not possess several insurance policies. If our goal in setting up simple theoretical models is to capture behavior in a positive sense, then such restrictions on preferences might be necessary. Of course, one can always argue that more restrictions belong elsewhere in our models, not on preferences. However, we are continually able to better understand the economic implications of higher-order risk attitudes, as set forth by Eeckhoudt and Schlesinger (2012). Explaining the rationale behind preference assumptions and restrictions should be an integral part insurance-decision modeling.

The single-risk model in a static setting as presented in this chapter should be viewed as a base case. Simultaneous decisions about multiple risky decisions as well as dynamic decisions are not considered in this chapter. Many extensions of this base-case model already are to be found in this Handbook. Certainly there are enough current variations in the model so that every reader should find something of interest. Of course, just as insurance decisions in the real world are not static, models of insurance demand should not be either. It is interesting for me to reflect on the knowledge gained since the first edition of this Handbook. I look forward to seeing the directions in which the theory of insurance demand is expanded in the years to come, and am encouraged to know that some of you who are reading this chapter will be playing a role in this development

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