Uncertain Bequest Needs and Long-Term Insurance Contracts[1]

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Abstract

We examine how long-term life insurance contracts can be designed to incorporate uncertain future bequest needs. An individual who buys a life insurance contract early in life is often uncertain about the future financial needs of his or her family, in the event of an untimely death. Ideally, the individual would like to insure the risk of having high future bequest needs; but since bequest motives are typically unverifiable, a contract directly insuring these needs is not feasible. We derive a long-term life insurance contracts that is incentive compatible and achieves a higher welfare level than the naïve strategy of delaying the purchase of insurance until after one's bequest needs are known. We also examine the welfare effects of our contract and we show how third-party financial products, although beneficial to the individual in the short run, can be detrimental to one's ex-ante utility.
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1 Introduction

People purchase life insurance to protect their dependents against financial losses caused by their deaths. Bernheim (1991) shows that there is a demand by breadwinners to hold part of their assets in a solely bequeathable form, as opposed to a form that could also be used for current consumption if one is alive. A similar result falls out of intertemporal equilibrium models, such as Farhi and Werning (2007), who show how it is desirable in an overlapping-generations context for parents to insure their children against the financial risks of their family.

In the life insurance market, most contracts extend many years into the future. Such prevalence of long-term contracts is largely explained in the literature as a way to protect against the "insurability risk." More specifically, a person's health status may deteriorate with consequences on the affordability of short-term life insurance in the future. In the extreme, one's health could deteriorate to such an extent that life insurance is unavailable. A long-term insurance contract with a front-loaded premium schedule can provide insurance against this reclassification or renewability risk.[1]

Although it may be advantageous from an insurability standpoint to arrange for life insurance early, the need for life insurance many years later depends on the future demographic structure of the household, as well as the financial condition of family members and their preferences, and may not be known in advance (see Lewis, 1989). Absent any insurability risk, it would therefore appear to be optimal to purchase life insurance contracts later in life, when bequest needs are better known. Another possibility is to purchase

[1]Such a risk arises even when one's health status is private information, as the market for short-term life insurance will then exhibit adverse selection. See Pauly et al. (1995). By contrast, if the change in insurability is contractible, it might be possible, at least in principle, to insure it directly in a manner similar to Cochrane (1995). For commitment problems associated with long-term contracting, see Hendel and Lizzeri (2003).

short-term contracts and to adjust the insurance level as needed at a later date.

In this paper, we show that a short-term purchasing strategy for life insurance is not optimal, even without the insurability risk. Intuitively, although delaying the purchase of life insurance can help individuals to determine the appropriate level of insurance, in concordance with their known bequest demand, one must still pay the extra insurance premium if one's demand turns out to be high. That is, one must plan for the possibility of needing to spend more on insurance premia in the future. Note that this form of "premium risk" has nothing to do with the insurability risk. Here the risk is on the budget required to finance the required amount of life insurance; not on whether or not the premium rate is higher.

We consider the design of a long-term life insurance contract that can help to partially insure the risk of possibly having a high bequest need in the future. Our model set-up is similar to that of Polborn et al. (2006), except that we first analyze a situation without insurability risk. When there is only a risk of demand type, the insurance premium per unit of coverage should not change for short-term contracts. Hence one can always buy more life insurance later at the same price. Polborn et. al. (2006) also mention this case of pure demand type risk, but conclude that there is no benefit to purchasing insurance earlier. The conclusion follows from their requirement of zero-profit insurance pricing within each period.

However, long-term contracts are not subject to this constraint, i.e., the zero-profit condition can be implemented over the duration of the contract, rather than within each time period. As a result, long-term contracts can allow individuals to partially hedge their future preference risk by effectively transferring some wealth from future states where their bequest needs are low to states for which bequest needs are higher. Although some family changes are observable, such as a change in the number of children, others are not. Our focus in this paper is this type of unverifiable shift in bequest demand. When bequest needs are not verifiable, the contract cannot just pay a transfer to anyone who claims to have high bequests needs. Hence, the long term contract is written with particular options, and the exercise of these options occurs via self-selection.

To be sure there are several reasons why an individual might wish to change his life insurance policy. Two potential reasons that have received some recent attention in the literature have been described as (1) the emergency fund hypothesis and (2) the policy replacement hypothesis. "The emergency fund hypothesis" was originally suggested by Linton (1932) and was brought into the modern literature by Outreville (1990). This rationale is based upon a liquidity need, such as when unforeseen medical bills require additional funding. In such a situation, the individual will reduce life insurance coverage because the funds are needed for other purposes. The other rationale for a change in life insurance is that a different life insurance contract would fit the changing bequest needs of the individual. This has been labeled "the policy replacement hypothesis" by Fier and Liebenberg (2012), who show that both hypotheses have empirical relevance.[2] Our analysis of long-term contracts works well for the policy replacement rationale, but not for the emergency fund rationale, as we explain in section 5 below.

Our analysis bears a similarity with the issue of flexible or refundable life

[2]Liebenberg, Carson and Dumm (2012) use dynamic data from the Survey of Consumer Finances to show that events that are empirically relevant in changing (or terminating) one's life insurance include adding children, becoming married, the recent death of one's spouse, and the sudden unemployment of the spouse. While the last of these events supports the emergency fund hypothesis, the first three are supportive of the policy replacement hypothesis. Fier and Liebebberg (2012), on the basis of a different data set of Americans over the age of 50, show that the determinants of lapse behavior change with the policyholders' age. For instance, a recent divorce is a significant determinant of lapse decisions only for the younger households in their sample.

annuities examined by Sheshinski (2007, 2010) and Direr (2010). Individuals can perfectly insure against the risk of longevity by purchasing deferred annuities early in life, but may not want to do so if they also face future liquidity risks, such as out-of-pocket health expenditures. Short-selling annuities (or borrowing against them) later in life subjects the individuals to adverse selection because they will by then have obtained better information about their longevity risk. This creates a type of "lemons problem" for the seller. As a result, individuals will be reluctant to convert assets into life annuities. Flexible annuity plans with withdrawal options are shown to mitigate this problem.

The longevity risk in these models is essentially the insurability risk from our life insurance model. In the annuity framework, flexible plans are useful because individuals need to insure against the up-dating of information about their longevity, i.e., individuals will be heterogenous in this respect at the time of the liquidity shock. Specifically, such plans mitigate the trade-off between the benefits of early contracting and the disadvantages in terms of one's reduced capacity to absorb expenditure shocks; they cannot insure individuals against liquidity risks per se. By contrast, in our framework, long-term life insurance contracts can provide insurance against uncertain future bequest needs even when there is no insurability risk.

In the next section, we set up our basic model, which excludes health risk and reclassification risk, so that we may focus solely on the riskiness inherent in bequest needs. We then examine a first-best world in which bequest type is verifiable. We describe the optimal insurance contract, which also provides protection against the risk of having a high bequest need in this setting. Next, we derive long-term life insurance contracts for the case where bequest type is unverifiable. These contracts are incentive compatible and achieve a higher welfare level than the naïve strategy of delaying the purchase of insurance until after one's bequest needs are known. These second-best contracts are also compared to the first-best case. We also explain why savings accounts, even if they include a penalty for early withdrawal, cannot replicate our second-best contracts, even though such contracts can effectively transfer some wealth between the high-needs and low-needs types.

We next discuss extensions and how our results relate to the literature. First, we extend the results to include a reclassification risk; in particular, a risk of having a higher probability of dying early. We characterize the optimal contract and examine some of its features. We then turn our attention to some relatively new third party financial products, especially so-called "life settlement" contracts. Such contracts can upset our long-term contract arrangement. Indeed, Polborn et al. (2006) find no role for long-term contracts as insurance against preference risk because they implicitly assume a type of actuarially-fair settlement market. We compare our results with those of Daily, et al. (2008) and of Fang and Kung (2010), both of which focus largely on reclassification risk. These papers also find that life settlement markets may be detrimental, but for a different set of reasons. Finally, we discuss more complex preference uncertainty whereby an individual might have uncertain needs even in survival states. In particular, we show which kind of preference uncertainty can (or cannot) be insured through long-term insurance contracts. This helps to facilitate a comparison between our results and those of Sheshinski and Direr concerning flexible annuity plans.

2 The Model

We develop a simple model of life-insurance purchases when individuals early in life are uncertain about their future bequest preferences. We compare two strategies: the purchase of death coverage through short-term contracts versus long-term contracts.

The market distinguishes between Term and Permanent life insurance.

Term contracts involve "pure insurance" and may be short-term (e.g., a period of coverage between one to five years) or very long-term (e.g., up to thirty years). Permanent life insurance is long-term and combines insurance and savings through policies that build up cash value. The insured is entitled to a cash surrender value upon cancellation of his policy. In our stylized model, long-term policies may be either of the Term or Cash Value kind.

There are two periods. Individuals in their early years (period 0) are uncertain about their bequest needs in later years (period 1). In a full fledged two-period set-up, individuals would face a risk of death in each period. Life insurance coverage through short-term contracts then involves the purchase of a policy for the risk of death in period 0 and, if one survives, the purchase of another policy for the risk of death in period 1. By contrast, a long-term contract purchased at the beginning of period 0 covers both risks and entails a contractually specified premium profile over time. To keep the model simple, however, we will abstract from consumption and risk of death in period 0. The distinction between short and long-term contracts nevertheless remains meaningful, as will become clear.

At the beginning of period 1, the individual learns of his preferences for bequest. At this information node, he can make decisions regarding life insurance, e.g., he can purchase a short-term insurance policy. Later in this period uncertainty about death is resolved: with probability p the individual dies, with probability $1 - p$ he survives.

The individual has initial wealth w_0 at the beginning of period 0. At this point, he may purchase a long-term life policy, even though he is yet unsure about his future bequest preferences. Such a policy specifies a death benefit and is financed by premiums paid in period 0 and in period 1. Alternatively, the individual may remain passive in period 0, i.e., he simply saves his initial wealth and waits until period 1 to buy life insurance through a short-term life policy after his bequest needs are known. To focus on preference risk, the probability of death p is non-random; thus, there is no premium risk regarding the future purchase of life-insurance should the short-term contract strategy be chosen.[3] For simplicity, we also assume that the interest rate for borrowing or lending is zero.

Denote by w^d and w^l the individual's final wealth at the end of period 1 in the states of death and survival respectively. Let i refer to the individual's type with respect to preferences for bequest. The beginning of period 1 or interim expected utility of final wealth is then

$$pv_i(w^d) + (1-p)u(w^l),$$

where $v_i(w^d)$ is the utility of leaving wealth w^d to dependents and $u(w^l)$ is the utility of wealth w^l in the state of living. Both functions are increasing and strictly concave. We further assume that $v_i'(w) > u'(w)$ for all w, implying a demand for life insurance. Taken together, $v_i(w^d)$ and $u(w^l)$ can be viewed as a state-dependent value function for the utility derived from the optimal consumption and savings strategies, given the individual's wealth in each state at the end of period 1 and taking implicitly into account the future labor income that a surviving individual would earn.[4]

Bequest needs are initially uncertain. In period 0, i.e., at the initial ex ante node, the individual does not know his bequest utility function, which

[3] In section 5, we extend the analysis to an uncertain mortality risk that depends on the individual's health status in period 1.

[4] In this setting, we can view w_0 as the present value of lifetime earnings for this person, whom for simplicity we will consider as the sole "bread winner" for the family. If the person dies in period 1, then family lifetime income will become less. The state-dependent utility v captures this income loss. Obviously, we are simplifying the basic insurance decision to a great extent. For example, we do not consider that future income might be risky, nor do we consider intermediate consumption. We also do not consider endogenous decisions about saving or alternative products such as annuities. See, for example, Campbell (1980). For a survey of many theoretical life insurance issues, see Villeneuve (2000).

can be either $v_B(\cdot)$ with probability π or $v_A(\cdot)$ with probability $1-\pi$. We assume that $v'_B(w) > v'_A(w)$ for all w, so that type B is the high-bequest type.[5] An individual's type, once learned, is private information, but insurers know the proportion of types in the population.

Any amount of life insurance coverage can be purchased in period 0, through a long-term contract, or at the beginning of period 1 through a short-term contract. Insurance markets are assumed to be competitive and there are no administrative costs; hence, we assume that the price of life insurance is actuarially fair.[6] The total fair premium for a policy with death benefit Q is pQ.

3 Short-Term Insurance Contracts

As a preliminary step, we examine the demand for life insurance when coverage is purchased only at the beginning of period 1 after the individual has learned his type. Even in a world with no insurability risk, there is a risk as to how much the total expenditure on insurance will be: from the perspective of period 0, the individual would like to insure against the risk of being a high-bequest type. In an ideal world, where bequest type is verifiable, this risk could be insured.

[5] Note that our assumption implies that B-types have a higher marginal utility at a *given* wealth level. *We assume that both types have the same starting wealth.* If high bequest-demand types referred to wealthier individuals, their observed marginal utility would obviously be lower than for low bequest-demand types. A more extreme formulation than ours would be $v_A(w) \equiv 0$, i.e., type A does not need death coverage. Preference uncertainty is discussed further in section 5 where we consider the possibility that the utility of survival wealth also depends on type.

[6] A zero-profit assumption is not crucial, but is fairly common in the literature. The basic result would not change if we assume that insurer's must make some fixed profit per contract.

The naïve strategy

The simplest strategy for buying life insurance is to wait until period 1. It is useful to characterize the demand for coverage as a function of some arbitrary wealth w at the beginning of period 1. Obviously, if nothing has been done before this date, then $w = w_0$.

For an individual with bequest type i and wealth w at date $t = 1$, the life-insurance objective is to

$$\max_{Q_i} pv_i(w - pQ_i + Q_i) + (1-p)u(w - pQ_i), \quad i = A, B.$$

The optimal coverage $Q_i^*(w)$ satisfies the first-order condition

$$v_i'(w - pQ_i^*(w) + Q_i^*(w)) - u'(w - pQ_i^*(w)) = 0, \quad i = A, B. \tag{1}$$

Risk aversion ensures that the second-order condition is satisfied. It is easily checked that $Q_B^*(w) > Q_A^*(w)$, i.e., B is indeed the high-bequest type.

Substituting for the optimal amount of coverage yields the optimal expected utility

$$\Lambda_i(w) \equiv pv_i(w - pQ_i^*(w) + Q_i^*(w)) + (1-p)u(w - pQ_i^*(w)), \quad i = A, B.$$

Here $\Lambda_i(w)$ is the value function for a person of type i who has wealth w at the beginning of period 1. Viewed from period 0 and treating bequest type as a random variable, $\Lambda_i(w)$ is a state-dependent utility function exhibiting risk aversion in each state of the world. To see this, apply the envelope theorem and use (1) to obtain

$$\Lambda_i'(w) = u'(w - pQ_i^*(w)), \quad i = A, B. \tag{2}$$

Since $Q_B^*(w) > Q_A^*(w)$, it follows that $\Lambda_B'(w) > \Lambda_A'(w)$. Differentiating a second time yields

$$\Lambda_i''(w) = u''(w - pQ_i^*(w))\,(1 - pQ_i^{*\prime}(w)) < 0.$$

The sign follows from

$$1 - pQ_i^{*\prime}(w) = \frac{v_i''}{(1-p)v_i'' + pu''} > 0, \tag{3}$$

where the expression is obtained by total differentiation of (1).

From (3), it is also easily verified that

$$1 - pQ_i^{*\prime}(w) + Q_i^{*\prime}(w) = \frac{u''}{(1-p)v_i'' + pu''} > 0. \tag{4}$$

Thus, bequest and net wealth in the survival state are normal goods, i.e., $w_i^l = w - pQ_i^*(w)$ and $w_i^d = w - pQ_i^*(w) + Q_i^*(w)$ are strictly increasing in the period 1 wealth w.

Insurance against bequest type

An individual who decides to wait until period 1 to purchase life insurance knows that he will purchase either $Q_A^*(w_0)$ or $Q_B^*(w_0)$, depending on his bequest needs. His period 0 ex ante expected utility is therefore $(1-\pi)\Lambda_A(w_0)+\pi\Lambda_B(w_0)$. Since $\Lambda_B'(w_0) > \Lambda_A'(w_0)$, transferring wealth at a fair price from the low to the high marginal utility state increases expected utility. Put differently, the individual would like to insure against the risk of being a high-bequest type.

Suppose for now that bequest types are verifiable. A contract could then be written in period 0 that pays some amount T at the beginning of period 1 if the person turns out to be type B. The fair premium for such a contract is πT paid in period 0. Wealth at the beginning of period 1 is now either $w_A = w_0 - \pi T$ or $w_B = w_0 - \pi T + T$ depending on the individual's realized bequest type, where $(1-\pi)w_A + \pi w_B = w_0$.

The optimal T^* solves

$$\max_T \, (1-\pi)\Lambda_A(w_0 - \pi T) + \pi\Lambda_B(w_0 - \pi T + T),$$

leading to the first-order condition

$$\Lambda_B'(w_0 - \pi T^* + T^*) - \Lambda_A'(w_0 - \pi T^*) = 0. \tag{5}$$

It follows trivially that $T^* > 0$, so that $w_B^* > w_0 > w_A^*$.

The life insurance purchased is then $Q_A^*(w_A^*)$ if needs are low and $Q_B^*(w_B^*)$ if they are high. The possibility of insuring against bequest needs yields a solution characterized by

$$u'(w_A^{l*}) = u'(w_B^{l*}) = v_A'(w_A^{d*}) = v_B'(w_B^{d*}), \tag{6}$$

where

$$w_i^{l*} = w_i^* - pQ_i^*(w_i^*),\ w_i^{d*} = w_i^* - pQ_i^*(w_i^*) + Q_i^*(w_i^*), \quad i = A, B.$$

We now essentially have a complete contingent claims market and equate marginal utility in all four possible states of the world. This is achieved by combining two types of insurance products: one insures against a premature death and the other insures against preference risks. Coverage against the risk of being the high bequest type is $w_B^* - w_A^* = T^* = p(Q_B^*(w_B) - Q_A^*(w_A))$, the difference in the life insurance premiums. Loosely speaking, T^* is the ex post transfer of wealth from state A to state B individuals. Condition (6) characterizes efficient ex ante insurance. We will refer to $(w_A^{l*}, w_A^{d*}, w_B^{l*}, w_B^{d*})$ as the first-best allocation.

Comparison

It is instructive to compare the first-best allocation with the outcome under the naïve strategy when preference risks are not insurable. In the first-best solution, wealth in the survival state is equalized across bequest types. Moreover, because of the wealth transfer and since bequests are normal goods, bequests are now larger in the high-bequest state and smaller in the low-bequest state, i.e., $w_B^{d*} > w_0 - pQ_B^*(w_0) + Q_B^*(w_0)$ and $w_A^{d*} < w_0 - pQ_A^*(w_0) + Q_A^*(w_0)$. Thus, the possibility of insuring against preference risks allows the bequest amount to more closely reflect needs.

– Fig. 1 and 2 about here –

Figures 1 and 2 provide a state-space representation of the consumer's problem at the beginning of period 1, when bequest type is known but one's date of death is still uncertain. In figure 1 preference risks are not insured. The negatively sloped straight line is the budget constraint arising from the insurer's zero profit condition:

$$(1-p)w_i^l + pw_i^d = w_0.$$

Indifference curves (iso-expected-utility) for both bequest types are shown. For bequest type i, the marginal rate of substitution between wealth in the surviving state and wealth in the death state is

$$-\left(\frac{dw_i^l}{dw_i^d}\right) = \frac{pv_i'(w_i^d)}{(1-p)u'(w_i^l)}. \tag{7}$$

For each type, the equilibrium occurs at the tangency point with the budget line, so that

$$-\left(\frac{dw_i^l}{dw_i^d}\right) = \frac{p}{(1-p)} \quad \text{or} \quad v_i'(w_i^d) = u'(w_i^l). \tag{8}$$

Because type B has steeper indifference curves, $w_B^d > w_A^d$ and $w_B^l < w_A^l$, leading to equilibria such as the contingent claims E_A and E_B in figure 1.[7]

In figure 1 we also represented the locus of type-dependent contingent claims satisfying (8). For a type-i individual, any pair on the type-i locus represents socially efficient insurance against the risk of death once bequest

[7] See Karni (1985) for a general treatment of models using such state-dependent preferences.

type is known. Accordingly, the locus will be referred to as the interim efficiency locus. Thus, the short-term contract insurance strategy entails interim efficient insurance against the risk of death. However, it does not yield ex ante efficient insurance.

Figure 2 illustrates the case where preference risks can be insured. The period 1 budget constraint is then

$$(1-p)w_i^l + pw_i^d = w_i, \quad i = A, B.$$

The equilibrium contingent claims in this case are E_A^* and E_B^*, with $w_B^{l*} = w_A^{l*}$ and $w_B^{d*} > w_A^{d*}$. Moreover, w_A^{d*} is smaller and w_B^{d*} larger than the corresponding amounts when preference risks are uninsured. The allocation satisfies the ex ante efficiency conditions (6). In particular, $\Lambda_A'(w_A) = \Lambda_B'(w_B)$, i.e., the marginal utility of wealth is equalized across bequest types.

Of course, direct insurance against bequest type is not feasible if one's bequest type is unverifiable. An individual purchasing such a policy would always want to claim that he is the high bequest type in order to receive the indemnity T^*. This is obvious from figure 2. Rather than staying at E_A^*, a type-A individual is better off claiming he is B and moving to the higher budget line.

4 Long-Term Contracts

We now turn our attention to the case where bequest type is private information and show how long-term contracts can nevertheless improve upon the naïve strategy of waiting until period 1 to purchase insurance. Note that it does not matter whether or not type is verifiable by insurers to implement the naïve strategy.

Many life insurance contracts include provisions that allow for changes to the contract at some future date, at the option of the insured. One such type of provision is an opting out opportunity: the insured can trade-in his policy at a later date at some pre-specified buy-back price, the so-called cash surrender value. Alternatively, a contract may not include a formal buy-back provision but the policy holder may simply lapse, i.e., he stops paying the extent premiums, thereby foregoing his right to the death benefit. We show that long-term insurance contracts, with either formal or informal opting-out provisions, can improve the individual's welfare when bequest types are non-verifiable. In particular, a well designed policy allows wealth to effectively be transferred from type-A individuals to type-B individuals.

Second-best arrangements

We first examine feasible allocations that do not depend on particular forms of contract. Consider the allocation that solves

$$\max_{w_A^l, w_A^d, w_B^l w_B^d} (1-\pi)\left[pv_A(w_A^d) + (1-p)u(w_A^l)\right] + \pi\left[pv_B(w_B^d) + (1-p)u(w_B^l)\right]$$

subject to

$$pv_A(w_A^d) + (1-p)u(w_A^l) \geq pv_A(w_B^d) + (1-p)u(w_B^l) \tag{9}$$

and

$$(1-\pi)\left[pw_A^d + (1-p)w_A^l\right] + \pi\left[pw_B^d + (1-p)w_B^l\right] \leq w_0. \tag{10}$$

A solution to this problem is an ex ante second-best optimum, given that bequest types are private information. The constraint (9) is the type-A self-selection condition stating that type A will choose the contingent claim (w_A^d, w_A^l) rather than (w_B^d, w_B^l); we abstract from the type-B self-selection condition because it will trivially be satisfied in the solution to the above problem. Condition (10) is the ex ante resource constraint in actuarial terms.

It is straightforward to characterize the solution to the above problem. The Lagrangian is

$$\begin{aligned}\mathcal{L} = (1-\pi)\left[pv_A(w_A^d) + (1-p)u(w_A^l)\right] + \pi\left[pv_B(w_B^d) + (1-p)u(w_B^l)\right] \\ + \mu\left[pv_A(w_A^d) + (1-p)u(w_A^l) - pv_A(w_B^d) - (1-p)u(w_B^l)\right] \\ + \lambda\left[w_0 - (1-\pi)\left(pw_A^d + (1-p)w_A^l\right) - \pi\left(pw_B^d + (1-p)w_B^l\right)\right],\end{aligned}$$

where μ and λ are nonnegative multipliers. From the first-order conditions, the solution $(\widehat{w}_A^d, \widehat{w}_A^l, \widehat{w}_B^d, \widehat{w}_B^l)$ is easily seen to satisfy:

$$(1-\pi+\mu)u'(\widehat{w}_A^l) = \lambda(1-\pi), \tag{11}$$

$$(1-\pi+\mu)v_A'(\widehat{w}_A^d) = \lambda(1-\pi), \tag{12}$$

$$(\pi-\mu)\,u'(\widehat{w}_B^l) = \lambda\pi, \tag{13}$$

$$\pi v_B'(\widehat{w}_B^d) - \mu v_A'(\widehat{w}_B^d) = \lambda\pi, \tag{14}$$

Clearly, $\lambda > 0$, i.e., the resource constraint is binding. The self-selection condition must be binding as well. If it were not, then $\mu = 0$ and the conditions would reduce to

$$u'(w_A^l) = v_A'(w_A^d) = v_B'(w_B^d) = u'(w_B^l),$$

which corresponds to the first-best allocation represented in figure 2. However, as already noted, type A strictly prefers E_B^* to E_A^*, implying that (9) is not satisfied. Thus, both multipliers are strictly positive; from (13), it also follows that $\mu < \pi$.

Combining (11) and (12) yields $v_A'(\widehat{w}_A^d) = u'(\widehat{w}_A^l)$; thus, type A obtains a contingent claim on his interim efficiency locus. Recalling that $v_B'(\cdot) > v_A'(\cdot)$, the conditions (13) and (14) yield $v_B'(\widehat{w}_B^d) < u'(\widehat{w}_B^l)$, implying that type B's contingent claim is below the interim efficiency locus for this type. Finally,

(12) and (14) are easily seen to imply $v'_B(\widehat{w}^d_B) > v'_A(\widehat{w}^d_A)$. Altogether, the second-best allocation therefore satisfies

$$u'(\widehat{w}^l_A) = v'_A(\widehat{w}^d_A) < v'_B(\widehat{w}^d_B) < u'(\widehat{w}^l_B). \tag{15}$$

The inequalities imply that the marginal utility of wealth for type B is always larger than for type A.

Proposition 1 *In a second-best allocation, (i) low-bequest types obtain interim efficient life insurance, while high-bequest types are over-insured against the risk of death; (ii) preference risks are only partially insured: from an ex ante perspective, individuals would wish that more wealth be transferred from bequest type A to type B.*

An illustration is given in figure 3. Interim "over insurance" by type B, represented by a point such as $\widehat{E}_B$ in figure 3, can be interpreted as a form of signalling cost. The allocation provides a larger bequest (and correspondingly smaller survival wealth) than type B would wish to purchase voluntarily on the basis of his implicit period 1 wealth level defined by $\widehat{w}_B \equiv p\widehat{w}^d_B+(1-p)\widehat{w}^l_B$. The intuition is that the "distortion" facilitates the transfer of wealth from state A to state B by making it more costly for type A to mimic type B.[8] By contrast, type A is efficiently insured on the basis of his implicit period 1 wealth level $\widehat{w}_A \equiv p\widehat{w}^d_A + (1-p)\widehat{w}^l_A$.

– Fig. 3 about here –

From the perspective of period 0, individuals are insufficiently covered against the risk of being the high-bequest type: $\widehat{w}_B > \widehat{w}_A$ but the wealth

[8] A separating menu is feasible that leaves both types on their interim efficiency locus. However, it is third-best and transfers less wealth between types. The second-best allocation trades off some distortion for type B against a larger transfer from type A to type B.

transfer from A to B is smaller than in the first-best, i.e.,

$$\widehat{w}_B - \widehat{w}_A < w_B^* - w_A^*. \tag{16}$$

To see this, note that

$$(1-\pi)\widehat{w}_A + \pi\widehat{w}_B = (1-\pi)w_A^* + \pi w_B^* = w_0.$$

If (16) does not hold, $\widehat{w}_A \leq w_A^*$ and $\widehat{w}_B \geq w_B^*$. A's period-1 implicit budget line in the second-best allocation would then be below the first-best one represented in figure 2, while B's budget line would be above the one in figure 2. Since $(\widehat{w}_A^l, \widehat{w}_A^d)$ is on the type-A interim efficiency locus, and recalling that bequest wealth is a normal good, we would therefore have $\widehat{w}_A^d \leq w_A^{d*}$. Now, B's contingent claim is given by the intersection of A's indifference curve through $(\widehat{w}_A^d, \widehat{w}_A^l)$ and B's budget line. Obviously, the foregoing would then imply $\widehat{w}_B^d > w_B^{d*}$. Altogether this would yield

$$v_A'(\widehat{w}_A^d) \geq v_A'(w_A^{d*}) = v_B'(w_B^{d*}) > v_B'(\widehat{w}_B^d),$$

which contradicts the second-best optimality conditions (15).

Implementation through opting-out contracts

Let the insurance contracts be defined by (P_0, P_1, Q, K) where P_t denotes the premium to be paid in period t, Q is the death benefit and K the surrender value, i.e., the price at which the policy can be sold back to the insurer. The premium P_0 is irrevocable because it is paid at the inception of the policy. However, at the beginning of period 1 the insured has the option to cancel the policy before the payment of P_1. If he does so, he is paid the contractual surrender value K.

Suppose that the contract is designed so that only type A cancels the policy. Recalling that π is the probability that the insured turns out to be type B, the insurer's non-negative profit condition is then

$$P_0 + \pi P_1 \geq (1 - \pi)K + \pi pQ$$

or equivalently

$$P_0 \geq (1 - \pi)K + \pi(pQ - P_1). \tag{17}$$

We assume that an insured can always drop his policy in period 1. This entails $K \geq 0$, otherwise a policy holder who wants to opt out would lapse without "claiming" the negative surrender value; in other words, a severance penalty $K < 0$ is deemed to be unenforceable.[9] Furthermore, individuals can always purchase insurance in the "spot market" through a short-term contract. This entails $P_1 \leq pQ - K$, otherwise P_1 would never be paid: it would be cheaper to surrender the policy, get paid K and purchase the coverage Q on the spot market for a premium equal to pQ. Combining the two inequalities, the contract must satisfy the no-commitment constraints

$$0 \leq K \leq pQ - P_1. \tag{18}$$

The right-hand side is the value of the policy at the beginning of period 1, i.e., the difference between the actuarial value of the face amount and the premium that remains to be paid. When the net value is strictly positive, P_0 on the left-hand side of (17) must also be strictly positive. The contract then involves front-loading in the sense that some payment is made up-front in period 0 for the future death coverage.[10]

[9] One-sided commitment (i.e., only the insurer is committed) is typical of long-term life insurance contracts. See Hendel and Lizzeri (2003) where this feature plays an essential role.

[10] In actual long-term policies, there is front-loading to the extent that the premium level early in the life of the policy is larger than the current actuarial cost of the death coverage (as in Whole Life or Level Term policies). Because we abstract from death coverage for period 0, our P_0 simply captures the amount of front-loading.

Insurers earn ex ante zero profit in a competitive market, so that (17) will hold as an equality at equilibrium. When only type-A individuals sell back their policies, the above contract will then effectively transfer the amount

$$T \equiv (pQ - P_1) - K \geq 0 \tag{19}$$

from type-A individuals to type-B individuals at the beginning of period 1. The transfer is the gap between the period 1 net value and the contractual surrender value. In essence, when $T > 0$, the insurer sells the face amount Q at a subsidized price. To see this, let (17) hold as an equality, solve for K and substitute in (19). Then $T > 0$ yields $P_0 + P_1 < pQ$. The insurer finances this subsidy by buying back his commitment at an unfair price from the low-bequest types.[11]

Such an arrangement works if appropriate incentive-compatibility conditions are satisfied. First, type-A individuals choose to sell back their policy and buy a new short-term policy on the spot market at actuarially fair prices.[12] Noting that the individual's wealth at the beginning of period 1 is $w_1 = w_0 - P_0$, the type-A self-selection constraint is:

$$\Lambda_A(w_0 - P_0 + K) \geq pv_A(w_0 - P_0 - P_1 + Q) + (1-p)u(w_0 - P_0 - P_1). \tag{20}$$

Secondly, type-B individuals must prefer keeping their policy:

$$pv_B(w_0 - P_0 - P_1 + Q) + (1-p)u(w_0 - P_0 - P_1) \geq \Lambda_B(w_0 - P_0 + K). \tag{21}$$

Finally, from the perspective of period 0, the arrangement must dominate the naïve strategy of waiting until period 1 to purchase insurance solely through short-term contracts:

$$U \geq (1-\pi)\Lambda_A(w_0) + \pi\Lambda_B(w_0), \tag{22}$$

[11] Writing (17) as an equality and solving for P_1 instead yields $K \leq P_0$. The difference $P_0 - K$ corresponds to the so-called *surrender charge* for accumulated life products.

[12] We make the usual assumption that an individual chooses the action designed for him when he is just indifferent between two courses of action.

where U denotes the expected utility provided by the long-term contract,

$$\begin{aligned} U \quad \equiv \quad & (1-\pi)\Lambda_A(w_0 - P_0 + K) \\ & + \pi \left[p v_B(w_0 - P_0 - P_1 + Q) + (1-p) u(w_0 - P_0 - P_1) \right]. \end{aligned} \tag{23}$$

In addition, the contract must yield a non-negative profit, i.e., condition (17). It is easily seen that the set of contracts that satisfy the foregoing constraints is not empty. In particular, they are satisfied by the contract defined by $P_0 = K = 0$, $Q = Q_B^*(w_0)$ and $P_1 = pQ_B^*(w_0)$ where $Q_B^*(w_0)$ is the optimal death benefit for type B under the naïve strategy. The non-negative profit condition and (21) then hold as equalities, and (20) as a strict inequality. Clearly, this arrangement yields the same outcome as the naïve strategy described in figure 1, implying that (22) is then satisfied as an equality.

As before markets are perfectly competitive and there are no administrative costs. In period 1, short-term policies can be purchased at an actuarially fair price for any amount of coverage. In period 0, long-term contracts are also offered. Insurers compete by trying to offer the "best" long-term contracts, where "best" means that the contracts maximize the purchasers' ex ante expected utility subject to the insurers earning non-negative profit. We now show that this can be done through an opting-out contract satisfying the no-commitment constraints (18). The argument is simply that the second-best allocation can be implemented by this form of contract.

Let the contract parameters (P_0, P_1, Q, K) be a solution to

$$w_0 - P_0 + K \quad = \quad \widehat{w}_A, \tag{24}$$

$$w_0 - P_0 - P_1 \quad = \quad \widehat{w}_B^l, \tag{25}$$

$$w_0 - P_0 - P_1 + Q \quad = \quad \widehat{w}_B^d. \tag{26}$$

where the right-hand side refers to the second-best allocation. Because the resource constraint binds in the second-best allocation (and recalling the definition of $\widehat{w}_A$), the above equalities imply

$$P_0 = (1-\pi)K + \pi(pQ - P_1), \tag{27}$$

meaning that the insurer earns zero profit provided type A opts out in period 1 while type B remains with the contract. If type A surrenders his policy, his period 1 wealth is $\widehat{w}_A$. Purchasing life insurance on the spot market at actuarial prices, his equilibrium prospect is then $(\widehat{w}_A^d, \widehat{w}_A^l)$ which he weakly prefers to $(\widehat{w}_B^d, \widehat{w}_B^l)$. Hence, type A opts out. By contrast, type B strictly prefers the prospect $(\widehat{w}_B^d, \widehat{w}_B^l)$ provided by the policy to the equilibrium prospect he would purchase with period 1 wealth $\widehat{w}_A$ if he were to opt out.[13]

The wealth transfer from type A to type B is

$$(pQ - P_1) - K = \widehat{w}_B - \widehat{w}_A > 0. \tag{28}$$

Therefore, any solution to (24), (25) and (26) with $K \geq 0$ satisfies the no-commitment constraints (18) and implements the second-best allocation. Observe that any such solution requires $P_0 > 0$.

Proposition 2 *An equilibrium opting-out contract yields the second-best optimum. The contract involves front-loading, it makes individuals strictly better off than the naïve strategy but remains second-best compared to the (symmetric information) case where bequest needs are directly insurable.*

Equilibrium contracts are not uniquely defined but they all implement the same allocation. It suffices that the contract parameters solve (24) to (26). For instance, there exists a second-best arrangement with $K = 0$. Such an arrangement can be interpreted as a contract without a formal opting out provision: the individual who wants to opt out simply lapses. The policy is

[13] The indifference curves satisfy the single crossing property. Hence, the type-B indifference curve through $(\widehat{w}_B^d, \widehat{w}_B^l)$ is above the budget line defined by the wealth level $\widehat{w}_A$. See figure 3.

then like Term insurance with lapse-supported pricing. Note that whether behavior is described as "opting out" or "opting in" is somewhat irrelevant. When there is no surrender value, the premium P_0 paid up-front can be interpreted as the price of a pure "option contract" giving the right to purchase the coverage Q at the price P_1. Equivalently, there are also second-best arrangements with $K > 0$ as in Cash Value insurance. The premium P_0 then contains a savings element: it must be large enough to cover, in actuarial terms, the cash surrender value paid to those who cancel their policy.[14]

The second-best allocation can also be implemented with other forms of long-term contracts. For instance, as with Universal Life Insurance, policyholders may be allowed to withdraw part of the cash value, thereby reducing the death benefit they are entitled to. In terms of our model, such a contract can be designed so that withdrawals appeal only to type A. Flexible long-term policies would of course be particularly relevant when there are more than two bequest types.[15]

We should point out that a second-best solution cannot be obtained through a savings strategy. Consider, for example, a savings account carrying wealth from period 0 to period 2. Such a savings account could be designed with a penalty for early withdrawal. This savings could be combined with a market for purchasing insurance in period 1, when an individual's type becomes known. Although such saving can succeed in transferring some wealth from the low needs type to the high needs type, the amount of insurance purchased will be efficient when viewed in period 1. In other words, it cannot be the second-best contract as depicted in figure 3. Thus, any such savings strategy can only be third best. As mentioned previously,

[14] The insurance industry also offers single-premium long-term contracts that are fully paid upfront (i.e., $P_1 = 0$ in our setup).

[15] The foregoing analysis is easily generalized to an arbitrary number of bequest types, assuming $v_i'(\cdot)$ is increasing in i, where $i = 1, ..., n$.

our second-best contract trades off some distortion (too much insurance in period 1) against a larger transfer from type A to type B. The long term insurance contract is a way to commit to overinsurance for those who do not cash out their policies early.

5 Discussion and Extensions

Contracts allowing some form of insurance against preference risks depend crucially on the self selection of types in exercising their options. But some relatively recent innovations in the financial marketplace may have an untoward effect on the feasibility of the long-term contracts we analyzed. In particular, the market for life settlements poses such an obstacle. Before discussing this issue, we show how our results extend to uncertainty about future health status. This will allow comparison with Daily et al. (2008) and with Fang and Kung (2010), who also argue that the emergence of life settlement markets may have negative welfare consequences. Finally, in the last subsection we reexamine our main point, namely that long-term life insurance contracts are useful when individuals benefit from the transfer of wealth from low to high bequest types. We discuss the extent to which this feature continues to hold under different forms of preference uncertainty.

Reclassification risk

Suppose that the mortality risk in period 1 depends on the individual's health status. With probability β an individual will turn out to be high-risk with death probability p_H, with probability $1 - \beta$ he will be low-risk with death probability p_L, where $p_H > p_L$. Risk types are learned only at the beginning of period 1 and then become common knowledge.[16] In period 0, individuals have the same expected death probability $\overline{p} = (1-\beta)p_L + \beta p_H$.

To start, let us abstract from preference uncertainty, i.e., all individuals have the same bequest utility of wealth function $v(\cdot)$. Waiting until period 1 to purchase life insurance through a short-term policy subjects individuals to a premium risk. Ex ante efficient insurance requires full coverage against this risk. The first-best death benefit Q^* is then easily seen to solve

$$\max_Q (1-\overline{p})u(w_0 - \overline{p}Q) + \overline{p}v(w_0 - \overline{p}Q + Q),$$

as if individuals faced the death probability $\overline{p}$ with certainty. Irrespective of the individual's risk category, final wealth is then $w^{l*} \equiv w_0 - \overline{p}Q^*$ in the survival state and $w^{d*} \equiv w_0 - \overline{p}Q^* + Q^*$ in the case of death, with $u'(w^{l*}) = v'(w^{d*})$.

Given the policy holders' inability to commit, efficient insurance can be provided by long-term policies only if contracts are sufficiently front-loaded (see Hendel and Lizzeri, 2003, and the discussion in Polborn et al., 2006). Otherwise, when coverage is not sufficiently prepaid, individuals who turn out to be low-risk will be tempted to drop their policy. In our set-up, for a Term contract with no surrender value, insurers earn zero profit when the premiums over both period satisfy $P_0 + P_1 = \overline{p}Q^*$. At the beginning of period 1, an individual has wealth $w_0 - P_0$. If he lapses, a low-risk type can then buy life insurance on the spot market at the premium rate p_L per dollar of coverage. If he remains with the policy, he is in effect exercising his option to purchase the coverage Q^* at the strike price P_1. This yields greater expected utility than lapsation provided $P_1 \leq p_L Q^*$, which requires $P_0 \geq \beta(p_H - p_L)Q^*$.[17]

[16] Due to medical screening, there is little asymmetric information at the underwriting stage in the life insurance market (see Cawley and Philipson, 1999). Polborn et al. (2006) study the case of adverse selection.

[17] Given zero profits, $P_1 \leq p_L Q^*$ is equivalent to $w_0 - P_0 \leq (1-p_L)w^{l*} + p_L w^{d*}$, where the

Hendel and Lizzeri (2003) provide empirical evidence for a model relating lapsation with the insufficient front-loading of Term contracts. The argument is that some individuals are credit-constrained early in life and must content themselves with insufficiently front-loaded policies. While such policies can put a cap on premiums for very adverse health events, they provide only incomplete protection against premium risk because of the possibility of lapsation by low-risk types (assuming there are several risk classes).

In our set-up individuals are not credit-constrained and they faced no reclassification risk. Nevertheless, due to preference shocks, some individuals will lapse under a Term policy. Similarly, some will terminate a Cash Value policy and cash-in the surrender value. Note that credit constraints early in life cannot be a major concern for individuals purchasing Cash Value contracts. We now complete our analysis by showing how long-term contracts can protect against both reclassification risk and preference risk.

If bequest types were verifiable, optimal ex ante insurance would fully protect against reclassification risk and also provide efficient protection against preference risk. The first-best allocation is then as depicted in figure 2 but with p equal to the average death probability $\overline{p}$. With unverifiable bequest types, by contrast, the second-best allocation $(\widehat{w}^d_{Aj}, \widehat{w}^l_{Aj}, \widehat{w}^d_{Bj}, \widehat{w}^l_{Bj})_{j=H,L}$ will provide only incomplete protection against either source of risk. In the appendix, we show that the second-best optimum satisfies

$$u'(\widehat{w}^l_{Aj}) = v'_A(\widehat{w}^d_{Aj}) < v'_B(\widehat{w}^d_{Bj}) < u'(\widehat{w}^l_{Bj}), \quad j = H, L, \tag{29}$$

together with

$$\widehat{w}^s_{AH} > \widehat{w}^s_{AL} \text{ and } \widehat{w}^s_{BH} < \widehat{w}^s_{BL}, \quad s = l, d. \tag{30}$$

Condition (29) is the same as our previous condition (15), but now for

right-hand side is the actuarial wealth of the low-risk type if he keeps the policy. Lapsation then yields a contingent claim on the efficiency locus that is "below" the prospect offered by the contract.

each risk type separately. The conditions (30) show that there is incomplete protection against health risk: a high-bequest type is better off if he is also a low risk type, while the converse holds for the low-bequest type. The intuition is that being both high-bequest and high-risk type is the worst event, in the sense that more wealth must be transferred to this event than to any other combination of types. The problem is that, while risk types are verifiable, bequest types must be induced to self-select. A low-bequest type then benefits from being high-risk compared to his alter ego low-risk type.

The second-best allocation can be implemented with long-term contracts of the form previously described, except that the period 1 parameters are now contingent on the risk-type, i.e., they are health-contingent. The contract is described by $(P_0, P_{1j}, Q_j, K_j)_{j=H,L}$. The conditions (30) then imply $P_{1L} < P_{1H}$ and $K_H > K_L$. As in so-called "renewable" life insurance contracts, an individual who keeps the policy will pay the smaller premium P_{1L} if he "qualifies", e.g., if a medical examination demonstrates that he is low-risk.[18] Conversely, the surrender value is partly indexed on the period 1 actuarial net value of the policy, implying that the high-risk type gets a larger surrender value.

Market for life settlements

The life settlements market, still primarily a US phenomenon, developed in the late 1990s as an extension of the market for "viaticals" which emerged in the 1980s.[19] A life settlement contract essentially offers to "buy back"

[18] There is of course a potential moral hazard problem in the decision to "qualify" policyholders. It may be mitigated by reputation concerns, e.g., insurance brokers inform new customers about insurers' record in this respect.

[19] Viaticals targeted terminally ill AIDS patients, while the settlements market concerns senior citizens with up to 15 years life expectancy. According to the Life Insurance Settlement Association, approximately $15 billions worth of life insurance policies were sold

the life insurance policy of an individual. This is effected via a third party paying cash to the insured, in exchange for being named the beneficiary of the life insurance death benefit and paying the premiums until the death of the original policyholder.

In a competitive settlement market and abstracting from transaction costs, an individual would be able to sell his policy for its intrinsic actuarial value. Ex post, given the type of contracts we have described, individuals with low-bequest needs will opt to sell the policy to a life settlement broker, rather than lapse or surrender the policy to the insurer, thereby receiving more money for their policy. The insured can then purchase a smaller insurance coverage at a fair price on the spot market.

The existence of such markets provides an alternative for the insured that is beneficial ex post, i.e., after signing the original long-term contract. Insurance companies had originally protested as these markets developed, claiming that they should have the exclusive right to buy-back (i.e., "settle") contracts that they had written. But others disagree. For example, Doherty and Singer (2002, 2003) tout the benefits of life settlement markets to the insurance consumer. Such analysis is incomplete, however, in that it excludes the fact that ex ante (i.e., prior to learning one's bequest type) one would prefer the longer term contracts described in this paper. A life settlement market will preclude such contracts from ever being offered. Although the long-term contracts we describe in this paper give the insurer monopoly power ex post, a competitive market ex ante should ensure that insurers cannot earn undue monopoly rents.

Daily et al. (2008) also argue that the secondary life insurance market may be detrimental, but for a different reason. They add uncertainty about life insurance needs to Hendel and Lizzeri's (2003) analysis of Term insurance, but without considering the possibility of insuring against preference

on the secondary market in 2006.

risk as such. In our notation, they assume $v_A(\cdot) \equiv 0$, i.e., type A no longer needs coverage. Absent a settlement market, type A individuals will lapse irrespective of their risk type. Given the insurers' ex ante zero profit condition, this prospect reduces the amount of front-loading required for a given degree of protection against premium risk. Introducing a settlement market, however, then increases the amount of front-loading needed. This in turn reduces the protection against premium risk when individuals are credit-constrained early in life. The reason more front-loading is required is that, ex post, only the high-risk types own a policy that has positive actuarial value on the secondary market. Hence, high-risk type A individuals, who would have lapsed in the absence of a secondary market, now find it advantageous to settle their policy, which therefore remains in force from the point of view of the primary issuer.

In these arguments, the detrimental effects of the secondary market are due to credit constraints and concern insurance against reclassification risk. In our analysis, the detrimental effect does not involve credit constraints (e.g., it would also arise under Cash Value policies) and it concerns insurance against preference risk. Type A individuals no longer lapse or surrender their policy because they get a better deal by settling the policy in the secondary market. As a result, the self-selection mechanism fails and it is no longer feasible to transfer wealth from type A to type B individuals.

Fang and Kung (2010) incorporate secondary life-settlement markets into their insurance-contract design. They derive the same detrimental effects of such secondary markets as in Daily, et al. (2008). However, they allow for the possibility of endogenous cash-surrender values, as an alternative to simply allowing for policy lapsation. If such cash-surrender values are allowed to be health contingent – dependent upon the policyholder's health at the time of surrender – there is still a welfare loss stemming from the existence of secondary markets, but the welfare loss is lower than it would be otherwise.

Our model shows that, even if health risks (i.e. reclassification risks) are absent, there is still a detrimental effect of secondary markets.

Preference uncertainty

Preference risks may take several forms. Our analysis involved "bequest types". We followed Polborn et al. (2003) in assuming that the types differed through their bequest utility of wealth functions, but had the same survival utility of wealth.[20] However, it may be that events affecting the utility of wealth in the death state also affect it in the survival state. For instance, when one's spouse becomes incapacitated, wealth as a bequest becomes marginally more valuable but so does the household's wealth in the survival state.

A more general specification would therefore be in terms of "preference types", where type $i = A, B$ is defined by both $u_i(\cdot)$ and $v_i(\cdot)$. As before, $v_i'(\cdot) > u_i'(\cdot)$ so that each type has a demand for life insurance. Type B is the one with high-bequest demand if

$$\frac{v_B'(w^d)}{u_B'(w^l)} > \frac{v_A'(w^d)}{u_A'(w^l)}, \quad \text{for all pairs } (w^d, w^l). \tag{31}$$

This is the single crossing condition depicted in figure 1. The demand for life insurance on the period 1 spot market, given wealth w, then satisfies $Q_B^*(w) > Q_A^*(w)$. Observe that this is not inconsistent with $u_B'(\cdot) > u_A'(\cdot)$ provided (31) holds.

However, condition (31) is not sufficient for our results. Long-term contracts were shown to be useful, for the purpose of insuring against preference risks, because they succeeded in transferring wealth from type A to type B. This is welfare improving only if the type with high bequest demand also is

[20] As mentioned above, Daily et al. (2008), as well as Fang and Kung (2010), adopt the same formulation, but with the extreme form $v_A(\cdot) \equiv 0$.

the one with "greater needs" at date 1. This property does not follow from (31) alone. Sufficient conditions are, for instance, $u'_B(\cdot) \geq u'_A(\cdot)$ together with (31).

In contrast, long-term life insurance contracts are not useful as protection against preference risk when the demand for bequest is negatively correlated with "needs". An extreme case, symmetrical to the one analyzed in this paper, is $u'_A(\cdot) > u'_B(\cdot)$ and $v_i(\cdot) = v(\cdot)$ for $i = A, B$. Condition (31) is satisfied, so that B is the type with high bequest demand, but this is due to A being the "high needs while alive" type. For instance, type A faces important out-of-pocket medical expenses necessary to maintain quality of life.[21] From an ex ante perspective, individuals would now want to transfer wealth from type B to type A. However, this cannot be accomplished through life insurance contracts because the relevant self-selection constraints cannot be satisfied. Indeed, abstracting from premium risks, the best long-term life insurance contract then does no better than the short-term contract strategy.[22] Liquidity or expense shocks can only be insured through other means, e.g., long-term health insurance aimed specifically at the expense shock.

It is worth emphasizing the preceding points. Opting-out of a long-term contract is consistent with self-selection only when an individual's wealth is reduced by opting out, otherwise everyone would do so given the existence

[21] To illustrate, let $u_B(w) = u(w)$ and $u_A(w) = u(w - m)$ where m is the expense or liquidity shock. This might be the case, for example, with viatical contracts, where a terminally-ill individual requires some immediate cash.

[22] In the second-best optimum, $(\widehat{w}_A^d, \widehat{w}_A^l)$ is on a higher budget line than $(\widehat{w}_B^d, \widehat{w}_B^l)$, with $\widehat{w}_A^d < \widehat{w}_B^d$. The binding self-selection constraint is now that of type B. This allocation cannot be implemented with life insurance contracts: B could pretend to be A, obtain $(\widehat{w}_A^d, \widehat{w}_A^l)$ and then purchase additional life insurance on the spot market, thereby reaching a prospect that is preferred to $(\widehat{w}_B^d, \widehat{w}_B^l)$. The possibility of purchasing insurance on the spot market now plays the same role as the possibility of selling one's policy on the settlement market, when the issue was to transfer wealth from A to B.

of spot markets for the purchase of short-term insurance contracts. This is precisely why our mechanism can still be useful when the high-bequest type is the one with "greater needs." It also illustrates why it does not work when it is the low-bequest type who has "greater needs".

It may be useful at this point to compare our results with those of Sheshinski (2007, 2010) and Direr (2010) in the context of flexible annuity plans. Define date 1 as the retirement date and let the individual's expected utility at that date be $u_j(c_1)+pv(c_2)$, $j = A, B$, where c_1 and c_2 denote consumption at date 1 and 2 respectively, and where p is now interpreted as the probability of survival up to date 2. At date 1, individuals learn their type, i.e., whether or not they face an expense shock, and they may also update their information about their survival probability p. In the absence of liquidity risks (i.e., $u_j(\cdot) = u(\cdot)$ for $j = A, B$), it is optimal for individuals to fully annuitize their retirement wealth early in life, say at date 0, thereby insuring against the risk concerning p. Conversely, if p is perfectly predictable but there is a date 1 liquidity risk, it is better for individuals to wait until date 1 before purchasing annuities. When p is non random, a flexible annuity plan purchased at date 0 can offer no improvement over this prospect. The reason is simply that, if withdrawals from the plan allow greater purchasing power, everyone would withdraw given that annuities can also be purchased at date 1.

6 Concluding Remarks

Uncertainty about future preferences arises in many contexts. This is true in particular for the purchase of life insurance. Indeed some 7.9% of long-term contracts involve lapsation (American Council of Life Insurers, 2009). Although the rationale for such a decision might be liquidity needs (the emergency fund hypothesis), it is often due to changing bequest needs (the policy replacement hypothesis). This paper has shown how, in this second circumstance, a long-term insurance contract can be designed to insure the uncertain future bequest needs of the individual. Our argument provides a rationale for a commonly observed feature of long-term life insurance contracts, namely that the contractual surrender value is less than the policy's cash value. Absent administrative costs, such a feature would not arise if long-term contracts were solely geared to protecting individuals against reclassification risks or to allow savings.

Obviously, we simplified the setting of our analysis by assuming away many complicating factors. This allowed our focus to be on the bequest needs and the probability of death. Integrating these results into more complex settings is difficult. Hopefully, our paper takes a good first step in this direction.

Appendix

We briefly characterize the second-best allocation when individuals are uncertain about their future risk and bequest types. Risk types are verifiable but bequest types are private information. Denote with β_j and π_i the probabilities of being risk type $j = H, L$ and bequest type $i = A, B$ respectively. In terms of the notation in the text, $\beta_H = \beta$ and $\pi_B = \pi$. Risk types and bequest types are independent events. The second-best problem of section 4 now becomes

$$\max_{w_{ij}^l, w_{ij}^d} \sum_{j=H,\,L} \sum_{i=A,\,B} \beta_j \pi_i \left[p_j v_i(w_{ij}^d) + (1 - p_j) u(w_{ij}^l) \right]$$

subject to

$$p_j v_A(w^d_{Aj}) + (1-p_j)u(w^l_{Aj}) \geq p_j v_A(w^d_{Bj}) + (1-p_j)u(w^l_{Bj}),\ \ j = H, L, \quad (32)$$

and

$$\sum_{j=H,\,L}\sum_{i=A,\,B} \beta_j \pi_i \left[p_j w^d_{ij} + (1-p_j)w^l_{ij}\right] \leq w_0. \quad (33)$$

The conditions (32) are the self-selection constraints for the low-bequest type in each risk class; as before, we abstract from the self-selection constraint for the high-bequest type. Condition (33) is the resource constraint across risk and bequest types.

Write the multipliers of the constraints (32) as $\beta_j \mu_j$, $j = H, L$ and let λ be the multiplier of constraint (33). Writing the Lagrangian, taking the first-order conditions and simplifying yields

$$\begin{aligned}
\pi_B v'_B(w^d_{Bj}) - \mu_j v'_A(w^d_{Bj}) &= \lambda \pi_B, && (34)\\
(\pi_B - \mu_j)u'(w^l_{Bj}) &= \lambda \pi_B, && (35)\\
(\pi_A + \mu_j)v'_A(w^d_{Aj}) &= \lambda \pi_A, && (36)\\
(\pi_A + \mu_j)u'(w^l_{Aj}) &= \lambda \pi_A, \quad j = H, L. && (37)
\end{aligned}$$

From (36) and (37), type A is on his interim efficiency locus, for each risk class. From (34) and (35), type B is below his interim efficiency locus, for each risk class. Complete insurance against risk class requires $\mu_H = \mu_L$, but it is easily seen that this is inconsistent with the self-selection constraints (32). Hence, $\mu_H \neq \mu_L$. One can show that $\mu_H > \mu_L$ is the only possibility consistent with B being below his interim efficiency locus (and therefore below that of A). The conditions above then imply $w^d_{AH} > w^d_{AL}$, $w^l_{AH} > w^l_{AL}$, $w^d_{BH} < w^d_{BL}$ and $w^l_{BH} < w^l_{BL}$.

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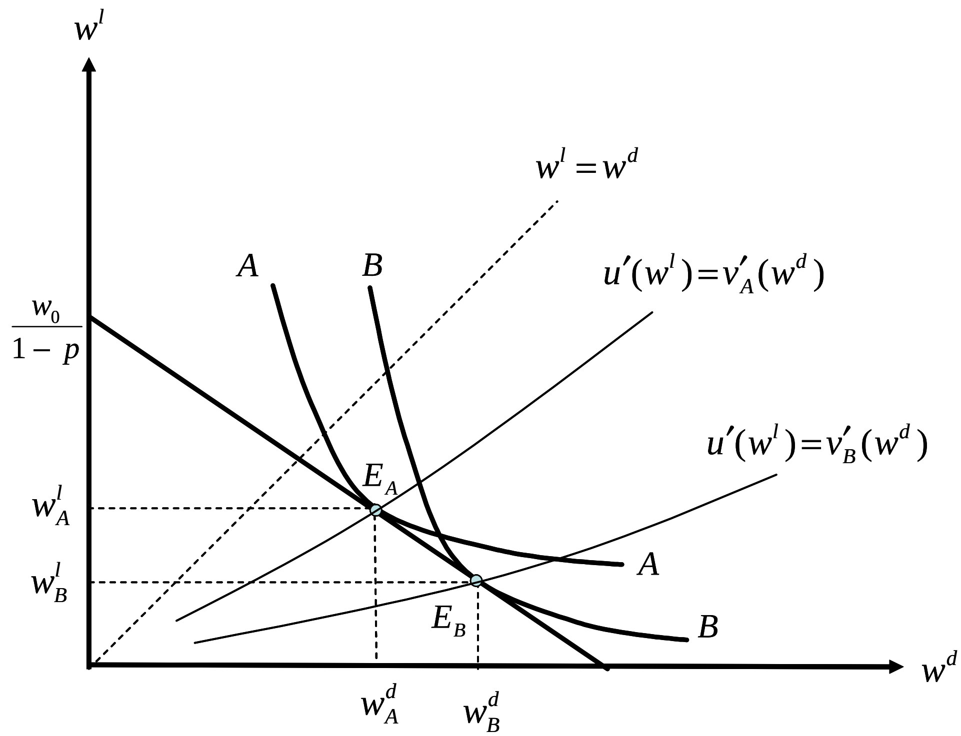

Figure 1: Purchase after type is known

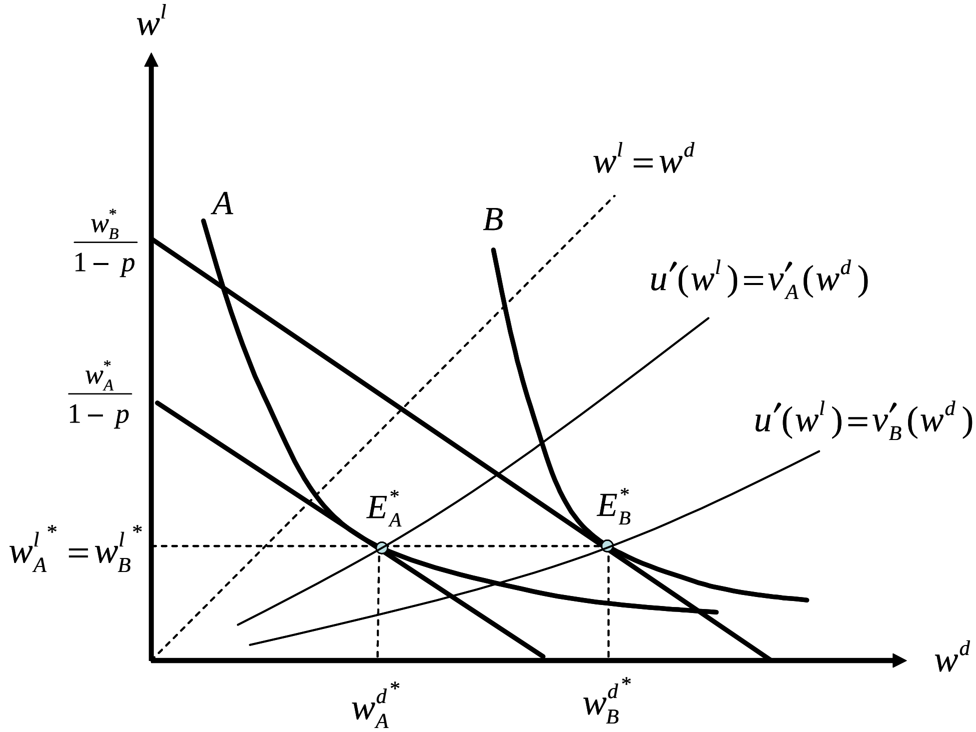

Figure 2: First-best insurance

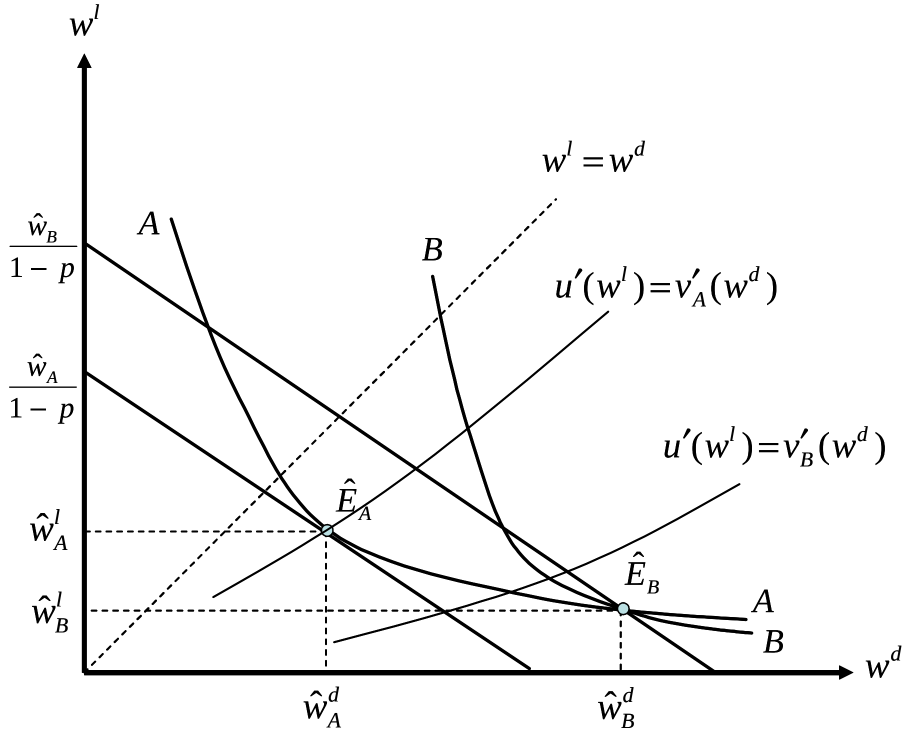

Figure 3: Second-best insurance